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# Truncated Sphere Noise Field Modeling for Acoustic Applications

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**Summary**
In many applications of beamforming and direction of arrival estimation, it is appropriate to assume that in addition to spatially white and directional interferences, there is an additional interference coming from an angular sector rather than a fixed angle. Examples of such noise arise in underwater acoustic and speech applications where an array of sensors is deployed to pick up a directional signal. While modeling and simulation of isotropic noise has been reported earlier, in this paper we try to develop a model for sector-based noise modeling, and simulation, where the noise field is assumed to be uniformly distributed over the surface of a sphere of a given solid angle specified by the limits of arbitrary azimuthal and elevation angles. The spatial coherence properties of such a noise field are explored and a method is proposed for its computation. The theoretical and computed results are seen to match very closely.

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## 1. Introduction

Noise field models are important to understand the performance of sensor arrays in applications like beamforming [1, 2] and direction of arrival estimation [3, 4]. A commonly used model found in the literature is the *"spherically isotropic"* or *"3D diffuse"* noise fields [5]. The noise sources in this model are assumed to be uniformly distributed on the surface of a sphere, i.e., the signals are incident on the sensors from all directions. A closed form solution for the spatial coherence function for collinear sub-arrays in spherically isotropic noise fields has been derived in [6]. The spatial coherence of noise fields evoked by continuous source distributed in a line is calculated in [7]. An exact series representation for a near field spherically isotropic noise model is introduced in [8]. Various algorithms have been presented in [5] to generate sensor array signals resulting from spherically isotropic noise fields.

The spherically isotropic noise field is not very useful to model more localized but diffuse sources of noise. In practical applications of microphone arrays used in speech applications, the noise field may arise, for example, from spatially extended sources like air conditioners, electrical machines, transformers, and spatially distributed crowds. Such sources are neither point-like nor a full sphere around the sensor array. They are better approximated as a truncated sphere or a sector of a sphere around the sensor array. The noise field resulting from a truncated sphere model will more closely represent the real life scenarios. In this paper, the noise sources are assumed to be uniformly distributed on a sector of sphere or truncated sphere. A mathematical expression of the spatial coherence function between two sensors is derived here for a truncated sphere noise field. The sensor array configuration is assumed to be linear. Theoretical and computed spatial coherence are compared. The method also leads to a simulation methodology for generating sensor signals incident on an array from such a noise field.

The remainder of the paper is structured as follows. Theoretical and practical computation of spatial coherence for truncated sphere noise field is taken up in Section 2. In Section 3 we compare the spatial coherence that results from the generated sensor signals from a truncated noise field with the theoretical spatial coherence. The effect of the presence of such a spatially extended source interference on the beamforming performance of an array is calculated and compared with the ideal performance of a non-extended directional interference to demonstrate the usefulness of such a simulation tool. Conclusions and future work are discussed in Section 4.

## 2. Spatial Coherence for Truncated Sphere Noise Field

Spatial coherence measures the correlation between waves for a given temporal frequency at different points in space. It is important in applications where antenna or sensor arrays are deployed - like mobile communication [9, 10], beamforming, blind source separation [11, 12] de-reverberation [13, 14, 15] and medical imaging [16].

### 2.1. Theoretical Formulation of Spatial Coherence

We consider here spatial coherence between two sensors located along the x-axis. We consider the noise sources to be contiguously distributed over the surface of a truncated sphere defined by $\phi_1$ to $\phi_2$ in azimuth and $\theta_1$ to $\theta_2$ in elevation. The radius $r$ of the sphere is assumed to be much larger than the distance $d$ between the sensors to assure that the wavefronts from each source appear to be plane at the sensor locations. Thus, the resulting noise field at each sensor is taken to be superposition of uncorrelated plane waves originating from various directions of the truncated sphere. The sensors are assumed to be omnidirectional. Any two sensor signals in space due to a plane wave arriving from an angle $(\theta, \phi)$ are related by

$$x_2(t) = x_1\left(t - \frac{\Delta}{c}\right) \tag{1}$$

where c is speed of sound and $\Delta = d\cos(\phi)\sin(\theta)$ [17] is the path difference of the plane wave arriving at the two sensors. As the auto correlation functions at the two sensors do not depend on path delay of the signal, the power spectral densities will be same at the two sensor positions, i.e.,

$$S_{x_1}(\omega) = S_{x_2}(\omega) \tag{2}$$

However, the cross-power spectrum density takes the form as

$$S_{x_1x_2}(\omega) = S_{x_1}(\omega)e^{\frac{-j\omega}{c}d\cos(\phi)\sin(\theta)} \tag{3}$$

The spatial coherence can now be computed by integrating the contributions over all plane waves [18] that originate from the truncated surface area A, i.e.

$$\gamma_{x_1x_2}(\omega) = \frac{\int_A S_{x_1x_2}(\omega)dA}{\int_A \sqrt{S_{x_1}(\omega)S_{x_2}(\omega)}dA} \tag{4}$$

where $dA$ is an infinitesimal area on the surface given by $dA = r^2 sin(\theta)d\phi d\theta$, as illustrated in Figure 1. Utilizing (2) and (3) in (4), the spatial coherence can be simplified as

$$\gamma_{x_1x_2}(\omega) = \frac{1}{A}\int_A e^{-j\frac{\omega}{c}d\cos(\phi)\sin(\theta)}dA \tag{5}$$

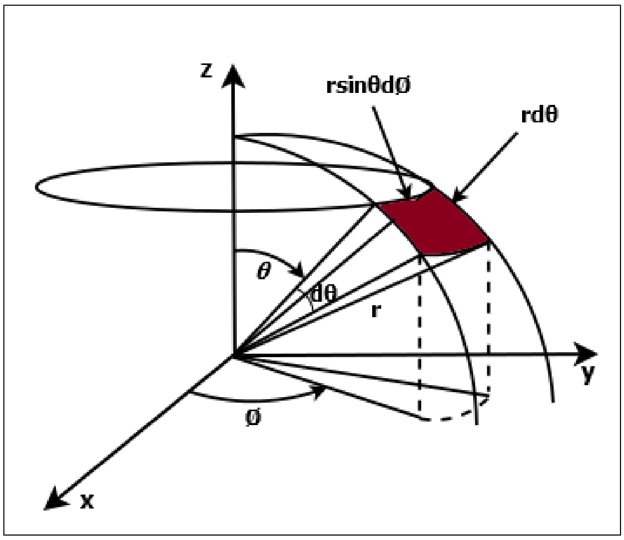


Figure 1: Spherical co-ordinate system with radius r, azimuth $\phi$ and elevation $\theta$.

Utilizing the expression for the area of the truncated sphere in (5), the spatial coherence for truncated sphere noise field is given by

$$\gamma_{x_1x_2}(\omega) = \frac{\int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2} e^{-j\frac{\omega}{c}d\cos(\phi)\sin(\theta)} r^2\sin(\theta)d\phi d\theta}{\int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2} r^2\sin(\theta)d\phi d\theta} \tag{6}$$

Thus the theoretical spatial coherence function for the truncated sphere noise field can be written as

$$\gamma_{x_1x_2}(\omega) = \frac{\int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2} e^{-j\frac{\omega}{c}d\cos(\phi)\sin(\theta)}\sin(\theta)d\phi d\theta}{(\cos(\theta_1) - \cos(\theta_2))(\phi_2 - \phi_1)} \tag{7}$$

where the numerator can be computed numerically.

### 2.2. Practical Computation of the Spatial Coherence

Equation (5) assumes continuous distribution of noise sources over the spherical sector. However in a simulation environment, only a finite number of discrete noise sources can be considered for generating the sensor signals from a distributed source. An important design issue to tackle here is to determine the number of discrete sources $N$ that would yield a good approximation to the theoretical spatial coherence function of the continuously distributed sources.

Assuming $N$ sources over the sector, the spatial coherence integral in (5) can be approximated by a summation as,

$$\hat{\gamma}_{x_1x_2}(\omega) = \frac{1}{N}\sum_{n=0}^{N-1} e^{-j\frac{\omega}{c}d\cos\phi_n\sin\theta_n} \tag{8}$$

where $N$ is the total number of noise sources that is assumed to be large for good approximation of the integral. $(\theta_n,\phi_n)$ is location of the $n^{th}$ noise source. The location is computed from the assumption of uniform distribution of the sources over the truncated sphere. For the uniform distribution of sources, the probability that a noise source exists in each infinitesimal area of a given value should be equal. More precisely, the probability of a noise source in the infinitesimal area $dA$ on the truncated sphere can be written as

$$P_r(\theta \leq \tilde{\theta} \leq \theta + d\theta, \phi \leq \tilde{\phi} \leq \phi + d\phi) = \frac{dA}{A}$$
$$= \frac{r^2 \sin\theta}{r^2(\phi_2 - \phi_1)(\cos\theta_1 - \cos\theta_2)} d\phi d\theta \quad (9)$$

Hence, the joint probability density function (pdf) of $\theta$ and $\phi$ can be expressed as

$$p_{\theta\phi}(\theta, \phi) = \frac{\sin\theta}{(\phi_2 - \phi_1)(\cos\theta_1 - \cos\theta_2)} \quad (10)$$

The joint pdf in (10) can be decomposed into two marginal pdfs given by

$$p_\phi(\phi) = \frac{1}{\phi_2 - \phi_1} \quad \text{and} \quad (11)$$

$$p_\theta(\theta) = \frac{\sin\theta}{\cos\theta_1 - \cos\theta_2} \quad (12)$$

The corresponding cumulative density functions (cdfs) can be written as,

$$P_\phi(\phi) = \frac{\phi - \phi_1}{\phi_2 - \phi_1} \quad \text{and} \quad (13)$$

$$P_\theta(\theta) = \frac{\cos\theta_1 - \cos\theta}{\cos\theta_1 - \cos\theta_2}, \quad (14)$$

respectively. So, the problem of generating sources with a uniform distribution over the spherical surface now reduces to that of generating locations $\theta$ and $\phi$ with densities given by (13) and (14), respectively. This can be easily done by generating independently generated uniform random numbers and transforming these to have the given distributions. Thus, taking $U = P_\phi(\phi)$ and $V = P_\theta(\theta)$ to be independent uniform random variable on [0,1], the expression for $\phi$ and $\theta$ can be parameterised as (15) and (16).

$$\phi = U(\phi_2 - \phi_1) + \phi_1 \quad (15)$$

$$\theta = \cos^{-1}(\cos\theta_1 - V(\cos\theta_1 - \cos\theta_2)) \quad (16)$$

More practically, we can vary U and V as $[0 : \frac{1}{N_\phi - 1} : 1]$ and $[0 : \frac{1}{N_\theta - 1} : 1]$ respectively to obtain $N = N_\phi N_\theta$ uniformly distributed source locations. The $N$ Gaussian sources at these locations can be considered to contribute to the truncated sphere noise field. The sensor signals can now be generated by superimposing the signals induced from each of these sources on a given sensor element, as proposed in [5]. A typical resulting spatial distribution of noise sources on truncated sphere is shown in Figure 2 for $\theta$ varying from $\theta_1 = 90°$ to $\theta_2 = 120°$ and $\phi$ from $\phi_1 = 0°$ to $\phi_2 = 180°$. Here the total number of noise sources $N = N_\phi N_\theta$ is taken to be 2048.

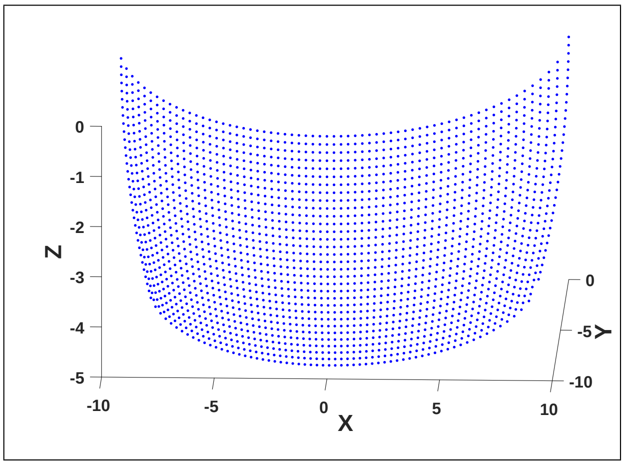


Figure 2: Uniformly distributed noise sources in a Truncated spherical noise field

## 3. Simulation Results and Discussion

### 3.1. Comparison between Theoretical and Computed Spatial Coherence

In this Section, theoretical and computed spatial coherence are compared. The total number of noise sources was taken to be $N = 512$ in a truncated sphere, defined by $\phi_1 = 20°$ to $\phi_2 = 80°$ in azimuth and $\theta_1 = 90°$ to $\theta_2 = 120°$ in elevation. Three microphones were placed in uniform linear array (ULA) configuration with the reference sensor at the origin of the truncated sphere. The distance between two consecutive microphones was 10 cm. The coherence between two sensor signals was estimated using Welch's averaged periodogram method [19] where fast Fourier transform was utilized with an FFT of length 256, and a Hanning window with 75% overlap was deployed. The theoretical and computed spatial coherence functions for $d = 10cm$ and $d = 20cm$ are illustrated in Figure 3. It is noted that the theoretical and computed spatial coherence functions match closely.

### 3.2. Spatial Coherence with varying sector Width

Spatial coherence between two adjacent microphones is analyzed with varying sector sizes in this section. For illustration, the sector width is varied along azimuth only with a fixed elevation. The sector size is varied in steps of $\Delta\phi = 20°$. The number of noise sources was taken to be $N = 64$ for all the sectors of different sizes. Without loss of generality, we consider that the source distribution is centered at $\phi = 90°$. The computed spatial coherence for different sector widths is shown in Figure 4. It is seen that as the sector size $\Delta\phi$ decreases, the spatial coherence increases. This is in line with the fact that if $\Delta\phi \to 0$, the source distribution converge to a point source. This results in a fully coherent sound field.

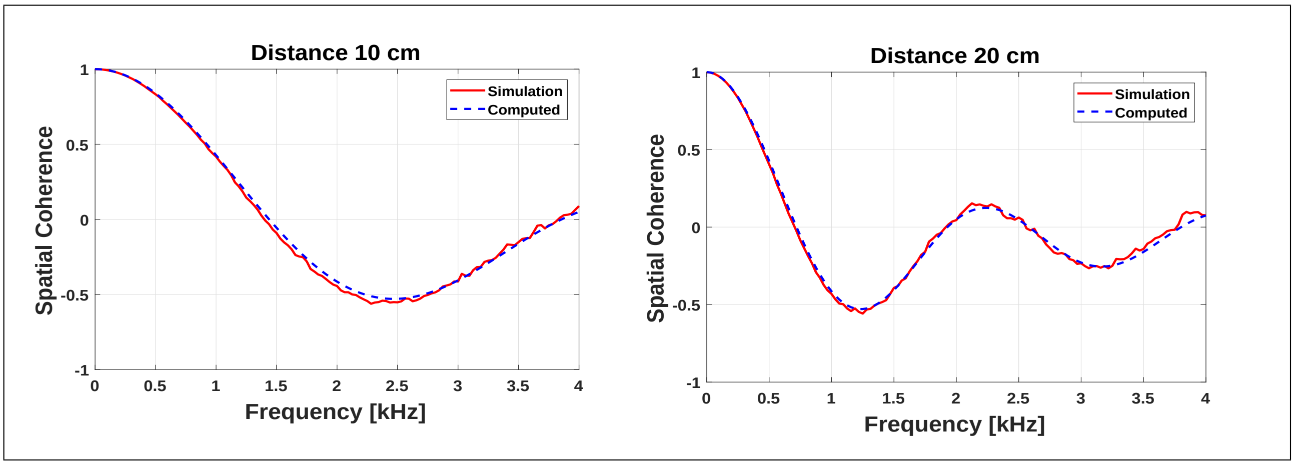


Figure 3: Spatial coherence between two sensors separated by 10 cm and 20 cm. Total number of noise sources was taken as $N = 512$ in a truncated sphere, defined by $\phi_1 = 20°$ to $\phi_2 = 80°$ and $\theta_1 = 90°$ to $\theta_2 = 120°$.

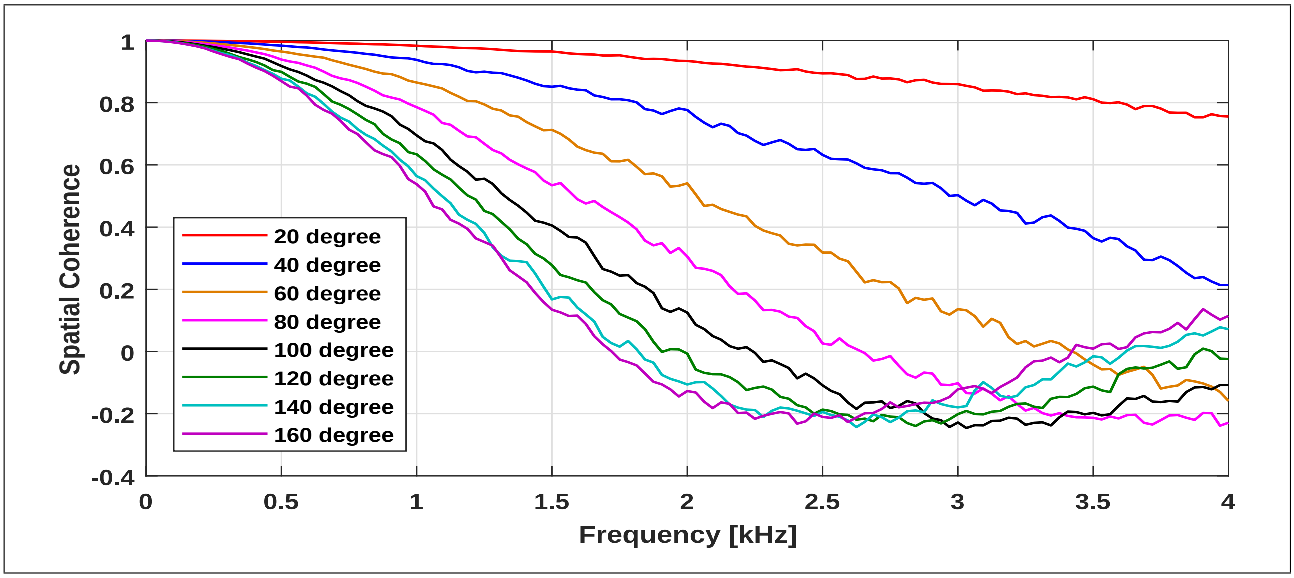


Figure 4: Spatial coherence between two sensors separated by $d = 10cm$ for different sector width $\Delta\phi \in (20°, 40°, 60°, 80°, 100°, 120°, 140°, 160°)$ centered at $\phi = 90°$.

### 3.3. Optimum Beamforming Performance with Varying Position of Extended Source

In this section, a possible application and usefulness of truncated sphere noise field modeling is discussed. In particular, effect of localized and sector based noise source distribution is investigated in the context of beamforming.

We assume that a target source is situated at 90° (broadside) direction. Performance of minimum dispersion distortionless response (MDDR) beamforming [20] is investigated here, in the presence of a localized and sector based noise sources. MDDR beamforming was chosen as it is a versatile beamforming method applicable even for non-Gaussian signals, and reduces to the more standard MVDR as a special case. Sixty four noise sources in azimuthal width of 20° were taken for the sector based interference. The localized interference and the center of the sector was varied between 10° to 170°. The input signal to interference ratio (SIR) was taken to be $0db$ for both the cases and the output SIR was observed.

The performance of the MDDR beamforming is plotted in Figure 5. It is noted that there is significant degradation in the output SIR when the sector based interference is used. Hence, the sector based noise/interference modeling will more closely reflect the actual performance when the actual scenario corresponds to an extended source model. As expected, the least output SIR is achieved when the interference direction matches with that of the target.

Table I: Objective parameter comparison for various type of interference

| $Typeofinterferene$ | $Directionofinterference$ | $outputSIR$ | $PESQ$ | $STOI$ |
|---|---|---|---|---|
| Localized interference | $70°$ | 16.14 | 4.02 | 0.99 |
| Spreaded interference | $65° - 75°$ | 15.13 | 3.09 | 0.96 |
| Spreaded interference | $60° - 80°$ | 12.97 | 2.3 | 0.91 |

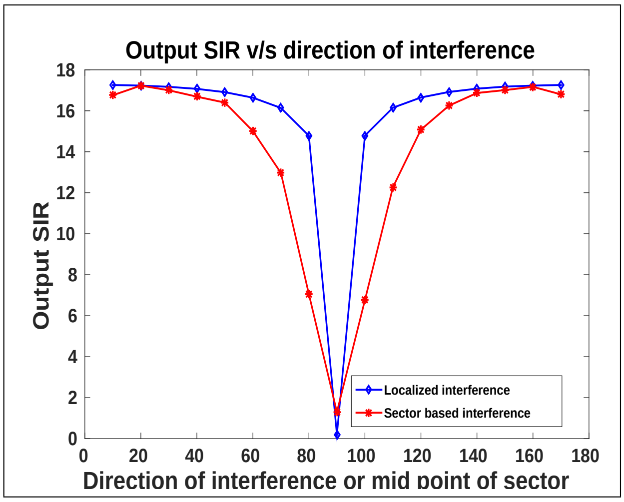


Figure 5: Beamformed output SIR v/s location of interference.

### 3.4. Optimum Beamforming Performance with Varying Sector Width

With the experimental conditions remaining same as in the preceding Section, beamforming performance evaluation is considered next for stationary interference with varying sector widths. The localized interference and the center of noise sector was taken at 70°. Hundred monte-carlo trials were run for a target source at 90° and input SIR $0db$. The performance of MDDR beamforming method in terms of output SIR, Perceptual Evaluation of Speech Quality (PESQ) [21] value and Short-Time Objective Intelligibility (STOI) [22] are presented in Table I. It can be seen that as the interference width increases, performance of the beamforming method degrades. This again reinforces the need of sector based modeling.

## 4. Conclusions and Future Scope

Truncated Sphere Noise Field Modeling is presented in this paper. The noise/interference sources are assumed to be incident from a solid angular sector. Spatial coherence formulation and computation is presented for such a noise field. Sector based noise source distribution is more practical for some real life applications. Application and importance of truncated noise field model is presented for optimum beamforming. Non-uniform spatial source distribution and non-Gaussian source amplitude distribution are currently being investigated.

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