Dissipative silencer performance with non-planar sources

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Summary
Measurements of silencer insertion loss are commonly taken with the assumption that the incident sound field is a plane wave. However this is unlikely to be the case across the entire frequency range of interest when using loudspeakers as a sound source. Instead it is likely that non-planar modes will be excited with a significant ratio of the total sound power. These non-planar duct modes are known to affect silencer performance; however this has not been studied in detail. This presents a significant commercial risk when designing silencers for large ducts. Standard test methods for the measurement of silencer performance do not identify non-planar excitations. Laboratory measured insertion loss may therefore be different from the performance under the expected plane wave source. The complex acoustic field within real-world sources, such as gas turbine exhausts or fans, further complicates design as the ratio of total sound power in each mode is unknown. Silencers in situ may therefore perform differently to silencers measured under laboratory conditions. The effect of non-planar modes is investigated here using measurements and predictions of silencer transmission loss. A method is presented whereby the sound power incident upon and transmitted by a silencer is determined from induct sound pressure measurements. Decomposing this data allows for the transmission loss to be calculated using the sound power in higher order modes. The equivalency of insertion loss and transmission loss metrics are demonstrated beyond the plane wave region when changes to the incident sound power are accounted for. Finite element methods are used to create numerical models of parallel baffle silencers to investigate the effect of non-planar sources. Predictions make use of the source characteristics measured during the experiment to demonstrate the importance of having knowledge of the source when designing silencers.

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1. Introduction
Noise control is an important consideration in the Power and Industrial sector given health and safety regulations and the regular proximity of industrial equipment to residential areas. The noise transmitted through ducts by the equipment must be kept below a specified level while also maintaining a low pressure drop. Failure to comply with these specifications, for example exceeding noise limits, can be met with strong penalties. However the competitive market means that over-engineering equipment is also discouraged as higher equipment prices can lead to losing a contract. Accurate prediction of sound propagation is therefore important to design an efficient noise control system.

The largest attenuation in the duct systems of large industrial equipment is typically provided by a silencer. Much research has been carried out to model silencers used in this environment. The efficiency of dissipative silencer cross-section has undergone much study with Nilsson and Söderqvist [1] presenting the bar silencer as an alternative to the simple parallel baffle silencer. Later Cummings and Astley [2] and Kirby et al. [3] compare the bar silencer to various other cross-sections and Yang et al. [4] further explore this concept for different bar shapes. Cummings and Astley [2] demonstrate the ability of the finite element method to determine the modal attenuation of a silencer but restrict their analysis to the least attenuated mode and discuss that this may affect accuracy. Kirby and Lawrie present a model combining the finite element analysis of the silencer cross-section to the inlet and outlet ducts by point collocation numerically demonstrating the effect of the fairings [5]. This

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model was later applied to show that the improvements to silencer performance can be illustrated by their modal attenuation [3].

The above work was carried out at room temperature with the main focus being HVAC systems. In large engine exhausts further conditions must be considered such as high temperatures and complex sound fields. The effect of temperature on dissipative silencers has been explored by Stevens and Ramakrishnan [6] showing significant changes to the spectrum as the peak insertion loss shifts to higher frequencies. Further work on the effect of temperature has been undertaken by Denia et al. for non-uniform temperature fields [7]. However there has been little work carried out concerning how non-planar sound fields alter silencer performance although it is widely recognised as being an important factor [2, 8, 5]. Mechel derived several theoretical multi-modal sound fields as approximations for unknown sound sources [8]. Kirby and Lawrie used these fields to demonstrate the gains to silencer insertion loss which are theoretically possible [5]. The changes presented are significant showing large improvements at mid and high frequencies of more than 10dB for each of Mechel’s theorised fields.

At the 4kHz limit of these predictions the improvement given through the contribution of the higher order modes accounts for more than double the insertion loss compared to the simple plane wave source. This is clearly an important mechanism which should be included in silencer calculation and would lead to reduced silencer lengths and optimised cross-sections.

It is difficult to take advantage of this fact for two reasons. Firstly, there is as yet no model which is validated against experimental data where the sound source is known and non-planar. So the ability of the model to accurately calculate the propagation of the higher order modes through the system is unknown. Secondly, an approximation for a complex industrial sound source is required as input. The work of Mechel [8] may be appropriate but again this has not been confirmed as no data exists concerning the mode power distribution of large industrial equipment, although some data is available for jet engines which share similarities to gas turbine engines used for power generation. In fact most measured silencer data is presented under the assumption of a plane wave source; the possible presence of higher order modes is either ignored or an attempt is made to reduce their effect as in the silencer testing standard ISO 7235 [9].

In this paper we will demonstrate that a numerical model based on the finite element method can accurately predict the insertion loss of a silencer subject to a non-planar source field. This will be accomplished by measuring the in-duct sound pressure before the silencer allowing for the propagating sound power to be calculated. Section 2 has a brief overview of the modelling techniques used in this investigation and the experimental set-up is discussed in Section 3.

2. Theory

In this work the finite element method (FEM) is used to model the propagation of sound through a silencer. The model has been described previously by Williams et al. [10] and so only an outline will be given here. The simulated system is divided into the inlet duct, $Ω_1$, silencer duct, $Ω_2 + Ω_3$, and outlet duct, $Ω_4$, see Figure 1. Sound pressure in the inlet and outlet ducts are expressed as a modal expansion

$$ p_I(x, y, z) = \sum_{m=1}^{M_I} (A_m e^{-ik\phi_m(x-x_s)}) + B_m e^{ik\phi_m(x-x_o)}\Phi_m(y, z), \quad (1) $$

$$ p_O(x, y, z) = \sum_{m=1}^{M_O} C_m e^{-ik\phi_m(x-x_s)}\Phi_m(y, z), \quad (2) $$

where $x_s$ is the plane of the source, $m$ denotes mode number, $M_I$ and $M_O$ the number of modes in the expansion for inlet and outlet ducts respectively, $A_m$, $B_m$ and $C_m$ are modal amplitudes, $k = 2\pi f/c$, $f$ is frequency, $c$ is the speed of sound, $\phi_m$ are normalised axial wave numbers, and $\Phi_m$ are eigenvectors of the empty duct. To calculate $\phi_m$ and $\Phi_m$ a numerical eigenanalysis is carried out on the surfaces $S_I$ and $S_O$ at $x_0$ and $x_1$.

In the silencer region the finite element method is used to model the propagation of sound. Use of this method is limited by computer resource as the 3D FEM quickly becomes expensive due to RAM usage as the number of nodes is increased; however it does allow for a detailed model of the silencer. Due to this limitation the model could only reach a sufficiently converged solution up to a frequency of 2kHz. Derivation begins from the unconvected wave equation:

$$ \frac{1}{c^2} \frac{\partial^2 p_I}{\partial t^2} - \nabla^2 p_I = 0 \quad (3) $$

![Diagram of the components of the model.](image)
where the pressure is

\[ p_q(x, y, z) = \sum_{j=1}^{M_q} N_{q_j}(x, y, z) p_{q_j}' \]  

(4)

\( N_{q_j} \) is the shape function and \( p_{q_j}' \) is the pressure at node \( j \). The derivation proceeds as shown in Williams et al. [10] to obtain a matrix equation which can be solved numerically using Matlab. Three-dimensional finite element meshes are generated using the Gmsh software [11]. Porous absorbent materials in region \( \Omega_S \) are modelled using the modified Delany and Bazley equations presented by Kirby and Cummings [12] using the material parameters from Williams et al. [13].

The acoustic response of the silencer is dependant upon the source applied and in this report two sources will be explored: the plane wave source and a multi-modal source field calculated from experimental data. The plane wave source amplitudes are set at

\[ A_{m=1} = 1 \]  
\[ A_{m>1} = 0 \]  

(5)

so that only the fundamental mode is excited.

To determine the modal amplitude of the experimental source measured sound pressure data is decomposed. The decomposition makes use of known sound pressure data transformed into the frequency domain by fast Fourier transform. Equation 1 can be written in matrix form for the pressure at several positions and rearranged in terms of the modal amplitudes to give

\[ C = W^{-1} P \]  

(6)

where \( P_n = p_{1,n}(x_n, y_n, z_n) \) is the \( n^{th} \) element of the matrix holding the measured pressure at position \( n \),

\[ C = \begin{bmatrix} A \\ B \end{bmatrix} \]  

(7)

is the matrix of modal amplitudes

\[ A = [A_1, \ldots, A_m, \ldots, A_{M_f}]^T \]  

(8)

and

\[ B = [B_1, \ldots, B_m, \ldots, B_{M_f}]^T, \]  

(9)

\[ W = \begin{bmatrix} W_A \\ W_B \end{bmatrix}, \]  

(10)

\[ W_{A,m,m} = \Phi_m(y_n, z_n) e^{-ik\phi_m(x_n-x_0)} \]  

(11)

and

\[ W_{B,n,m} = \Phi_m(y_n, z_n) e^{+ik\phi_m(x_n-x_0)}. \]  

(12)

Using Equation 6 the modal amplitudes \( A \) can be calculated and used as the input modal amplitudes for the model.

The in-duct sound power can be calculated for each mode as

\[ W_{A,m} = \frac{3}{2\rho c} |A_m|^2 \int_{S_I} |\Phi_m|^2 dS \]  

(13)

where \( \rho \) is the density of air. Transmission loss is then calculated by

\[ TL = -10\log_{10} \left( \frac{\sum_{m=1}^{M_f} W_{C,m}}{\sum_{m=1}^{M_f} W_{A,m}} \right). \]  

(14)

3. Experiment

The experiment is set-up as in Figure 2. At one end of a duct a loudspeaker acts as a sound source while the other exits into the room. A duct lined on all four sides with a length of 900mm and material depth of 200mm is placed after the sound source to decouple the source from the test duct. The test duct has a rectangular cross-section 800mm across by 600mm high with a total length after the lined duct of 4360mm and is made from 2mm thick steel. The test rig is located inside a large room where the walls are lined with insulation to reduce reflections.

Sound pressure is measured across five planes: one plane external to the duct and four in-duct planes. The measurements taken outside of the test duct measure the pressure transmitted by the duct so that insertion loss may be calculated. This surface is a hemi-cylindrical surface \( 1m \) from the outlet. Measurements are taken at 105 positions by 7 microphones over 15 test runs. A reference microphone measures sound pressure within the duct to verify the repeatability of the sound source between test runs. Sound pressure is measured across planes 1, 2, 3 and 4 within the test duct at distances of 650mm, 1050mm, 3470mm, and 3870mm after the lined duct respectively. Measurements are taken over each cross-section at 64 positions by 8 microphones during 8 test runs. Each test run consists of taking a measurement of the sound pressure level for a period of 36s at a sampling rate of 25.6kHz with source noise set as a 12s logarithmic sweep between 20Hz and 11200Hz.

A parallel baffle silencer was tested in this experiment. Three 100mm wide baffles were placed across the duct with a 83mm airgap between the side baffle’s and the duct wall and a 166mm airgap between each baffle. Each baffle was 595mm high with a total length of 960mm. This left a gap at the top of each baffle of approximately 5mm to allow the baffles to be moved into and out of the duct. The baffle frame was made of three vertical, non-perforated steel channels and two horizontal, non-perforated steel channels with each channel being 30mm wide. The vertical channels
create a solid front and rear fairing with a solid separation plate half way along the baffle. The horizontal channels enclose the baffle at the top and bottom. This creates two volumes that are exposed to the duct airway at the sides of the baffle covering these openings. Perforated plates with an open area of 32% are attached to the sides of the baffle. The total perforated length of each baffle where the internal volume is exposed to the airway is 870mm with a height of 535mm. The baffles are placed 2360mm from the end of the lined duct so that they are 1310mm after plane 2 and 150mm before plane 3. The baffles are packed with sheets of rock wool at a density of 100kgm$^{-3}$ with a flow resistivity of 51400Nsm$^{-4}$ [13].

4. Results and Discussion
In this section measured insertion loss, measured transmission loss, and predicted transmission losses are presented for the parallel baffle silencer. The method by which transmission loss is measured is only valid up to 1700Hz where there are 40 modes propagating in the duct. There are less modes decomposed than theoretically possible as the microphones were not arrayed in a rectangular grid pattern due to experimental limitations. Data for the transmission loss will only be presented up to this frequency limit. This of course removes the 4kHz and 8kHz octave bands from the analysis, where the effect of higher order modes on transmission loss is significant. However the first cut-on frequency is at 214Hz and so there remains a large bandwidth over which higher order modes can propagate and pressure can be decomposed over.

Figure 3 presents the measured insertion loss and transmission loss of the silencer showing that they are similar up to 1700Hz. There is an oscillation in the insertion loss across this frequency range primarily caused by the change to the incident sound power between the set-up with and without the silencer installed. A phenomenon that has been reported previously by Roux et al. and demonstrated up to 800Hz using the two microphone method [14]. This oscillation has a small wavelength and so does not significantly affect octave band data after averaging, however it is noticeable in this data due to the use of a high frequency resolution. The error caused by the change to incident sound power is problematic as it will vary between test facilities and noise sources changing the measured insertion loss and decreasing the reproducibility of measured data. Here the change to incident sound power is less than ±1dB above 250Hz, however changes of ±6dB have been observed without the use of the lined duct/modal filter section installed; the modal filter therefore serves a purpose in decoupling the sound source from the test duct. The insertion loss can be corrected to remove this error using the difference between the sound power incident on the test duct with and without the silencer installed improving the agreement with transmission loss. In doing so it can be shown that the assumption of equality of insertion loss and transmission loss is valid if there is constant power so that the transmission loss of the model can be compared to the insertion loss of the experiment. In the data that follows this corrected insertion loss will be compared to the numerical transmission loss.
Figure 4. Performance of silencer 1. Measured insertion loss with incident power correction, predicted with plane wave, predicted with measured source power.

The predicted transmission loss of the silencer using a plane wave sound source generally compares well against the experimental data, see Figure 4. There are small under predictions below 450Hz giving an error of 1dB, however above this the prediction diverges and a difference of 2-3dB is found; this is expected from previous work where calculations are found to be inaccurate across the peak insertion loss [3]. Analysis of silencer data shows that there are dependencies on the material type and silencer length. Aside from the differences in amplitude the most notable difference is the lack of peaks in the prediction. There are clearly several strong features in the measured data such as the peak at 950Hz and across the region from 1250Hz to 1700Hz; using the plane wave source these features are not captured. For the silencer example presented in this paper the practical implications of this inaccuracy are small. The maximum error is approximately 3dB at these frequencies and this is an acceptable margin of error in an engineering application as it can be accounted for by the safety margins placed on equipment. However larger errors have been observed both for the under prediction at low frequencies and across the peak attenuation of the silencer. Further to this the multi-modal nature of the sound field will continue up to high frequencies where they dominate performance. Correspondingly the peaks observed at mid-frequencies are also observed at high frequencies but will not be predicted by the calculation. This leads to an under prediction by the plane wave source model in the 4kHz and 8kHz octave bands which is a region that regularly limits silencer design and results in designs that are over engineered.

Figure 5. Ratio of total sound power incident upon silencer 1. (0,0) plane wave mode, (even,even) modes, remaining modes.

Kirby et al. [3] hypothesized that the errors may be caused by inaccuracies in the theoretical model, for example the material model or behaviour under multi-modal conditions. They also discussed that the equivalency between insertion loss and transmission loss may be a possible source of error but this has been shown to be valid within Figure 3. The effect of the non-planar source on model accuracy can be explored here using the decomposition of in-duct sound pressure.

To show the characteristics of the sound source the calculated ratio of the sound power incident on the si-
lencer in the (0,0) plane wave mode, the (even, even) modes, and all remaining modes compared to the total sound power is illustrated in Figure 5. The decomposition shows that there is significant energy outside of the plane wave mode over a large bandwidth. Below 430Hz there is negligible power outside of the plane wave however above this frequency an increasing ratio of the total sound power propagates within the higher order modes and by 1325Hz the plane wave contains less than 50% of the total sound power; this ratio continues to fall up to the frequency limit of the data. The change to the sound power at 430Hz is caused by an increase to the ratio of sound power in the fourth duct mode which has the indices (2,0). The (2,0) mode cuts on initially at 429Hz holding negligible power and increases to hold 10% of the total power at 467Hz. Subsequent symmetric modes carry more of the sound power reducing the plane wave further. Given that the duct of the test rig is approximately symmetrical across the central lines of the cross-section it is expected that there will be negligible energy in the modes with odd indices. However the power in the asymmetric modes was not negligible with over 20% of the energy held by these functions. This may be caused by errors in the set-up or in the in-duct measurements.

While the lined duct is useful in decoupling the sound source from the test duct it was initially installed to act as a modal filter according to the guidelines within ISO 7235 [9]. This silencer testing standard calls for the modal filter to have a minimum attenuation of 3dB in the low frequency region and 5dB above the cut-on frequency of the duct. Testing has shown that the lined duct met this requirement up to the 4kHz octave band; however as illustrated in Figure 5 the majority of the sound power is not within the fundamental mode above 1200Hz. Meeting the requirements of the standard has therefore not been sufficient for ensuring a plane wave dominant incident field showing that more care must be taken than implied by the standard when designing the modal filter. It is suggested that the modal sound power distribution should be measured during experiments to allow for more data regarding the field exciting the silencer although this becomes infeasible at high frequencies.

The measured source amplitudes are applied to the source of the numerical model with the result plotted in Figure 4. The effect is relatively minor for this silencer below 780Hz where the plane wave continues to dominate attenuation. However above this frequency the predicted transmission loss begins to closely mirror the features of the measured data. For example the peak at 950Hz and those between 1250Hz and 1700Hz are well captured demonstrating that the lack of detail when using the planar field was caused by an inaccurate source assumption. This provides confidence that the model will also predict the peaks measured in the 4kHz and 8kHz octave bands given the correct source. Developing a source excitation similar to that of a complex industrial sound source would allow for these peaks to be accounted for increasing the performance in the high frequency octave bands. Higher noise limits could then be met with smaller, more efficient silencers.

Despite the improved agreement an over prediction remains between 450Hz and 1250Hz and so this cannot be explained by the source model and a different explanation is needed for the errors observed over this frequency range. It is hypothesized that this error is caused by the use of an inaccurate material model. For example the equivalent fluid model used here may not be sufficiently detailed for the silencers explored. Evidence supporting this can be found within the work of Nennig et al. on the use of the Biot theory [15]. Nennig et al. observe a difference between the attenuation of a limp material model and full Biot model similar to the difference between prediction and measurement presented here. That is the Biot model shows an increased transmission loss at low frequencies, but then a decreased transmission loss before tending back towards the limp model.

5. CONCLUSIONS

Silencer performance has been investigated for many of the conditions likely to be found within large industrial equipment but there is little work on multi-modal source fields. This subject is of interest due to the potential benefits which can be realised with minimal changes needed to existing designs. Towards this end the modal power distribution in the duct before the silencer is measured for a loudspeaker noise source. The ratio of power in the fundamental plane wave mode is found to quickly fall with increasing frequency until the higher order modes hold the majority of energy so that the test duct is exposed to a non-planar field. The shift towards this complex sound field is not prevented by the use of a modal filter complying with the requirement set by ISO 7235 with the plane wave mode energy becoming negligible at the frequency limits. Gathering this power data allowed for an accurate non-planar source data to be applied to a numerical model.

Insertion loss and transmission loss is measured for a silencer with dimensions relevant to HVAC applications and also large industrial ducts. Installation effects are observed to create a deviation between insertion loss and transmission loss when the sound power incident on the test duct changes between configurations with the silencer installed and uninstalled. This error is reduced by decoupling the test duct from the sound source by, for example, introducing a lined duct section. Accounting for this error shows that the metrics are approximately equal, validating comparisons between measured insertion loss, which is easier to measure, and calculated transmission loss.
A 3-dimensional finite element prediction generated using the measured source data is found to correctly predict the features of the measured spectrum when the source sound power data is used. While the measured data extended only up to 1700Hz the ability to predict the peaks will extend up to higher frequencies given the correct theoretical source. The use of a multi modal field as opposed to the conservative, relatively risk free, plane wave source is expected to deliver increased silencer performance. However an approximation of a gas turbine exhaust must be developed to make use of this when designing noise control equipment.

References


