Determination of nonlinear acoustic properties of perforates using band-limited random excitation

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Summary
Perforates are used for noise control in automotive mufflers and aircraft engines as well as for other vehicles and machines. Their properties and noise reduction are known to depend on the mean flow field and other parameters such as temperature and acoustic excitation level. It is therefore of interest to understand how the properties of perforates varies with the level of acoustic excitation. Methods for studying nonlinear harmonic interaction effects, for perforates, using single tone excitation and Poly-harmonic distortion models or nonlinear scattering matrices has been studied. These techniques typically require measurements with a number of different acoustic loads. It would be more attractive to directly be able to extract the nonlinear acoustic properties from a more limited set of experiments using either random or periodic excitation. Multi input – single output techniques for nonlinear system identification using broadband random excitation has been tried with limited success. One reason is the mixing of the sound pressure signal incident from the acoustic source with the sound pressure transferred to higher frequencies by nonlinear effects at the perforate sample. The present paper is an attempt to combine band-limited broadband excitation with nonlinear scattering matrices describing the nonlinear transfer of energy to higher frequencies. By analyzing acoustic energy transfer at frequencies at least two to three times higher than the high frequency cut-off for the bandlimited broadband random signal it may be possible to extract information to estimate the parameters of the non-linear scattering matrix.

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1. Introduction

Reduction of noise and emissions in flow ducts is of importance in industrial applications such as vehicle exhaust systems and aircraft engine liners. A strategy for controlling noise is to use perforate plates to create losses. The acoustic properties of these perforates depend on the mean flow and other parameters e.g. temperature and acoustic excitation level. There have been many studies on the acoustic impedance of perforates [1-17], including non-linear properties. More specifically in number of papers experimental techniques for determining harmonic interaction effects for perforates under non-linear conditions have been developed [1-7]. Experiments were made using both pure tone and random excitation and the relevant parameters controlling the non-linearity were discussed. In [1], a study of harmonic interaction effects using two-tone excitations was made and in [2], the study was extended to multi-tone excitation for different types of perforates. In [3], a study was made of using non-linear system identification techniques for this purpose. The effect of sample non-linearity when performing impedance tube measurements were studied in [4], along with an outline of multi-port techniques for characterization of samples with non-linear properties. These multi-port techniques were further developed and experimentally tested in [5-7]. By reviewing the pervious information, it is obvious that many investigations of non-linear effects occurring when high amplitude sound waves are incident on perforated plates or orifice plates have been published, see e.g., [8-16]. Generally, they all agree that the non-linear losses are associated with vortex shedding at the outlet side of orifice or perforate openings. That is the reason the nonlinear multi-port techniques with sinusoidal excitation developed and tested in [4-7], aimed at taking non-linear energy transfer between sound field harmonics into account.

The methods for studying nonlinear harmonic interaction effects, for perforates, using single tone excitation [4-7], typically require measurements with a number of different acoustic loads. It would be more attractive to directly be able to extract the nonlinear acoustic properties from a more limited set of experiments using either random or periodic excitation. In [17], multi input – single output techniques [18-22], for nonlinear system identification were tried without much success. The idea of treating a nonlinear path as a separate non-linear input after which system identification is performed as for a linear two input one output system was first introduced by Bendat [18-20]. The general methodology, for arbitrary nonlinear systems, as used in this paper was first published by Rice and Fitzpatrick [21]. The techniques have later been summarized by Bendat [20]. An example of a more recent work applying a modified version of the technique to mechanical system is [22].

In the present paper a method for extracting nonlinear scattering matrix data according to [5-6], however, by using broadband random excitation is attempted. The background is an impedance tube test from [6], comparing impedance results obtained when there was random noise excitation in the whole frequency range up to 2000 Hz with results obtained if there was only excitation up to 500 Hz. Figure 1 shows the real part of the normalized impedance, where the black solid line curves were obtained using broadband excitation covering the whole frequency range while the red dashed line curves were obtained with excitation only up to 500 Hz. The explanation for the results obtained at high levels of excitation is that transfer of energy to higher frequencies gives a sound source located at the sample surface. The sound field from this source will interact with the sound field from the loudspeaker source.

The two-microphone wave decomposition technique used in impedance tube measurements [23-26], assumes that we know where the source is located relative to the microphones and the sample under test. If the sound source is located at the sample, but we assume that the sound source is at the loudspeaker side, we will as have discussed in [6], get a result corresponding to the impedance looking back into the impedance tube from the sample but with the wrong sign. It can be seen that due to the non-linear energy transfer to higher frequencies a result is obtained also in the frequency range where there is no excitation. If you change the sign this result look like what you expect to see for an impedance measurement in an open-ended pipe. With random excitation in the whole frequency range we get a mix of the impedance caused by the direct excitation from the loudspeaker and the impedance results caused by the non-linear energy transfer giving a sound source at the sample. It is therefore not possible to get a result characterizing the non-linear sample using random excitation.

It can also be noted from Figure 1 that the resistance for the case with excitation up to 500 Hz becomes negative at around 800 Hz indicating that the source is at the sample, giving sound generation instead of dissipation. The explanation is that the level of excitation caused by non-linear energy transfer increases with increasing frequency because of an accumulation effect where more frequency components from below 500 Hz can contribute to a specific frequency component as the frequency increases.

The methods developed in [4-7], for single tone excitation are based on the assumption, backed by experimental evidence, that nonlinear energy transfer at the sample mainly occurs from lower to higher frequencies and to odd harmonics of the excitation frequency.

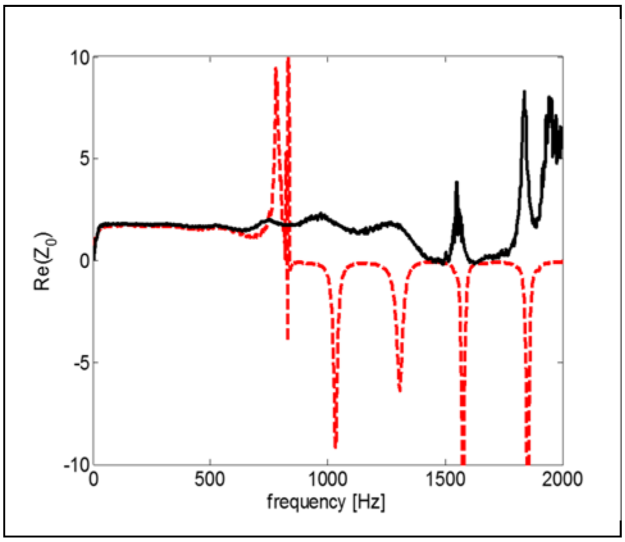

Figure 1. Real part of normalized impedance for a perforate sample with 2% porosity, 1 mm hole diameter and 2 mm thickness with 116 dB sound pressure level and 0.012 m/s particle velocity at the sample: — — — broadband random excitation up to 2000 Hz, ——— broadband random excitation up to 500 Hz.

The strongest coupling is between the excitation frequency and the third harmonic. The idea tested in the present paper is that if bandlimited random excitation is used, for instance excitation only below 500 Hz such as in Figure 1 this information can be used to study nonlinear energy transfer to frequencies three times as high. For instance, using broadband excitation at 300 Hz and studying the transfer of energy to 900 Hz, assuming that the only signal occurring at 900 Hz is caused by nonlinear energy transfer at the sample.

In the first section of this paper non-linear system identification has been described. In the second part the experimental setup and results will be introduced. In third section the results are shown. Finally, there is a section on conclusions and future work.

2. Nonlinear system identification

As described in the previous section the purpose of this study is using band-limited random exaction to study nonlinear energy transfer to frequencies three times higher than the excitation frequency, using one-sided multi-port techniques as described in [6]. Figure 2 shows a sketch of one-sided multi-port of an impedance tube with the perforates sample mounted at the one end. Moreover, a loudspeaker on the left side provide acoustic excitation.

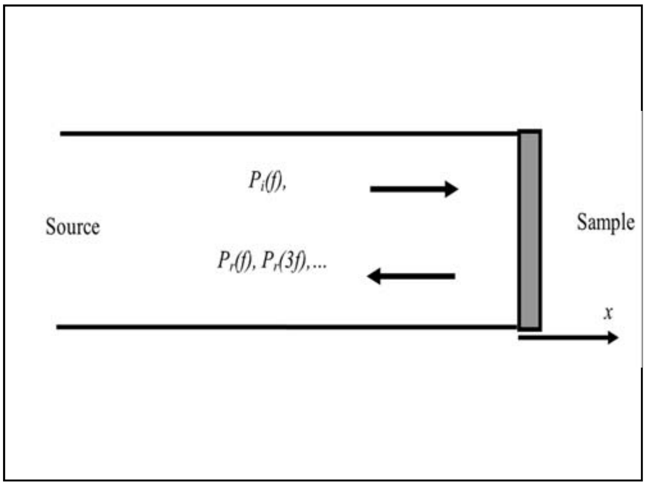

Figure 2. Pressure waves in a test duct terminated by a non-linear sample.

The Study of harmonic interaction effects used in the present paper require some assumptions and simplifications. First it has been assumed that the signals are analytical. Second, the nonlinear energy transfer only occurs from lower frequency components to higher frequency harmonics. Third, the nonlinear energy transfer is only to odd harmonics. Finally, there is only one frequency component with high level excitation and the system components for other frequencies can be determined from linear scattering matrix or reflection coefficient measurements.

With these assumptions and simplifications, the relation between the high-level excitation at frequency f and response at f and 3f can be described by the following matrix equations for a one-port, such as in the impedance tube,

$$\begin{pmatrix} P_r(f) \\ P_r(3f) \end{pmatrix} = \begin{bmatrix} S_{11}(|P_i(f)|) & 0 \\ S_{31}(|P_i(f)|) & S_{33}(|P_i(f)|) \end{bmatrix} \begin{pmatrix} P_i(f) \\ P_i(3f) \end{pmatrix}, \qquad (1)$$

Where: P_i and P_r are the incident and reflected waves (towards and away from the sample) and S_{33} is measured through a separate low-level excitation measurement at frequency 3f while the other two components are measured with varying level of excitation at frequency f. There is only excitation at frequency f so the pressure waves travelling in positive x-direction at higher frequencies are caused by reflection at the source.

3. Experimental technique and setup

An impedance tube was in the experimental tests used to perform the wave decomposition and obtain the pressure wave amplitudes P_i and P_r. Three B&k ¼-inch microphones were used giving the smallest microphone separation 5 cm and the largest 30 cm. This made it possible to cover a frequency range from 60 Hz up to the maximum frequency used in the experiment 2000 Hz [24,25]. The sample was placed in a holder at the end of the duct and measurements were made with and without the sample. Time domain data was collected using random excitation. Tests were made for a number of different perforate samples with varying porosity, hole diameter and hole thickness. The results presented here are for a sample with 2% porosity, 1 mm hole diameter and 2 mm thickness. Microphone separation 5 cm and the largest 30 cm. This made it possible to cover a frequency range from 60 Hz up to the maximum frequency used in the experiment 2000 Hz [24,25]. The sample was placed in a holder at the end of the duct and measurements were made with and without the sample. Time domain data was collected using random excitation. Tests were made for a number of different perforate samples with varying porosity, hole diameter and hole thickness. The results presented here are for a sample with 2% porosity, 1 mm hole diameter and 2 mm thickness. The signal processing used to perform wave decomposition, that is to extract the pressure wave amplitudes P_i and P_r from the measured pressure signal signals P_1 and P_2 can normally be made using standard techniques [22,25]. It is described in some detail here since the procedure may have an influence on the relative phase relation between the excitation and the higher harmonics. This is the procedure used sofar:

1) We have measured the pressure at two microphone positions P_1 and P_2.
2) Power spectral densities and cross power spectral densities have been determined using Welch technique averaging. The number of averages was 400.
3) A relative calibration has been applied to compensate for amplitude and phase differences between the microphone measurement channels.
4) The pressure wave amplitudes (P_i and P_r) are calculated using standard equations.

This procedure works fine in the linear case when there is no coupling between frequency components. For the nonlinear case there is a risk that the Welch technique averaging can cause a loss of phase information between the fundamental excitation frequency and higher frequency components, in the present case frequencies three times higher than the excitation frequency.

4. Experimental results and discussion

Measurements were made with 10 different levels of excitation and either with broadband random excitation up to 2000 Hz or up to 500 Hz. In both cases data was taken with a sampling frequency of 5120 Hz giving a maximum analysis frequency of 2000 Hz. In the analysis broadband random data covering a frequency range from f = 265 Hz such that 3f = 795 Hz, to f =500 Hz such that 3f = 1500 Hz, was used. Data presented here were determined with analysis bandwidths $\Delta f = 2.5$ Hz which gives results for 95 frequencies. To illustrate the nonlinearity data will be presented either as a function of the absolute value of the amplitude of the incident pressure wave at the sample (Abs(P_i(f))) or as a function of an inverse Strouhal number (St) proportional to the particle velocity (u(f)) at the sample.

$$1/\mathrm{St} = \mathrm{u(f)}/\omega/\mathrm{t}\,, \qquad (2)$$

where ω is the angular and t the thickness of the perforate. The inverse Stouhal number is equivalent to the ratio between particle displacement and perforate thickness. In some cases, results will also be presented as function of frequency.
Figures 3 and 4 shows the absolute value of $S_{31}$from Eq. (1) which is related to the transfer of energy from the fundamental frequency to the third harmonic. It can be seen that there is a scatter in these results both when plotted against inverse Strouhal number and absolute value of the incident pressure wave amplitude This may have to do with

the issue of phase relations between the excitation frequency and the higher harmonic discussed in the previous section. This issue is under investigation. Also, this figure shows that the absolute value of the S_{31} for low level of excitation is almost zero and for the

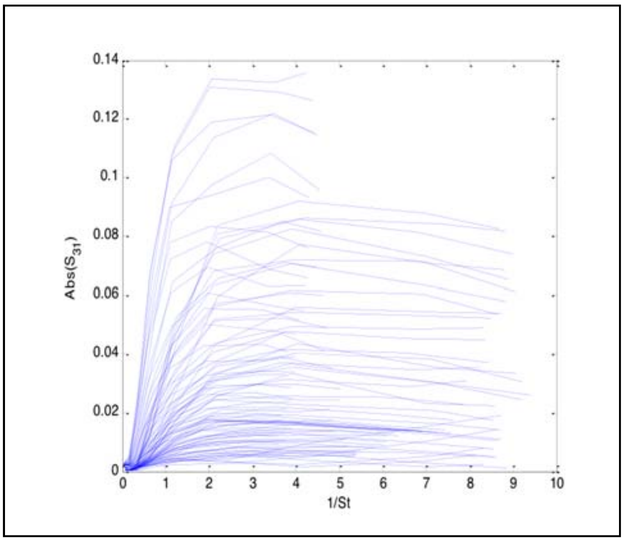

Figure 3. Absolute value of S_{31} according to Eq. (1). Plotted against inverse Strouhal number.

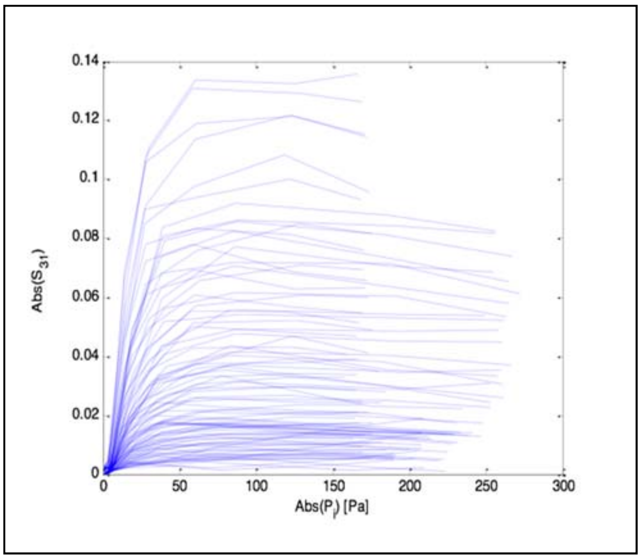

Figure 4. Absolute value of S_{31} according to Eq. (1). plotted against absolute value of the incident pressure wave.

high level of excitation it is constant. Figure 5 shows the maximum of the absolute value of $S_{31}$for each frequency from fs =265 Hz to F =500 Hz. This shows that there is a clear frequency dependence but not a trend such as increase or decrease with increase in frequency. The cause of his frequency dependence is under investigation and will be part of future studies.

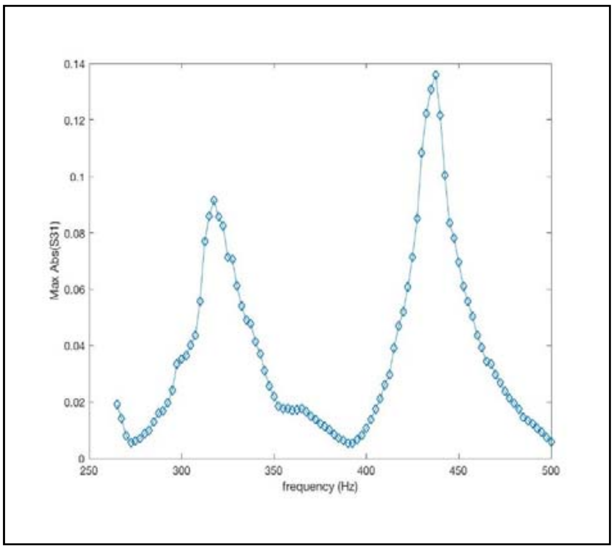

Figure 5. Max absolute value of S_{31} Plotted against frequency (Hz).

5. Conclusions

Experimental methods for determining nonlinear acoustic properties of perforates used in automotive mufflers and aircraft engine liners have been discussed. A new idea where nonlinear scattering matrix data, which has previously only been obtained with tonal excitation, is measured using bandlimited broadband random excitation has been tested. There are still issues with this technique which are under continued investigation, but the method of determining nonlinear scattering matrix data from bandlimited broadband random excitation instead of from single tone excitation is potentially promising. Further studies will include comparison with tonal excitation covering the same frequency range and investigations of methods for preserving the phase relation between the excitation frequency and higher harmonics.

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