Estimation of coupling power proportionality for the SEA of highly dissipative dynamical system in medium-high frequency domain

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Abstract

Until nowadays, an energetic approach called Statistical Energy Analysis (SEA) is widely used in acoustical prediction. However, this method depends a lot on important coefficients such as Coupling Power Proportionality (CPP), which is usually difficult to be analytically identified. For decades, people have worked on the extensions or some alternative models of SEA, and some of them are trying to give an analytical expression of CPP. Nevertheless, it is always a challenge to deal with highly dissipative materials. In this paper, based on SEA principles, we propose a quasi-analytical solution of CPP, which circumvents the limitation of light damping assumption. With this energetic method, the Sound Transmission Loss (STL) of a dynamical multilayer system can be predicted correctly and rapidly, regardless of the dissipation of each layer. The numerical result of this proposed method is compared with direct analytical methods and an alternative model of SEA for validating.

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1. Introduction

Nowadays, the analysis of buildings acoustic performances becomes more and more difficult due to their structural complexity and efficient numerical models are required to predict their acoustic insulation. Finite Element Method (FEM) [1] is well known for its high accuracy and wide range of applications in the low-frequency domain. However, its numerical cost is very expensive when the modal density number is getting high such as in the medium- and high-frequency domains. In high frequency domain, an energetic approach called Statistical Energy Analysis (SEA) [2, 3, 4, 5, 6, 7] is widely used and allows the vibro-acoustic behavior of complex structures to be predicted. Nevertheless, it should be noted that the SEA does not only rely on important coefficients, such as the Coupling Power Proportionality (CPP), which are usually difficult to be analytically identified, but also relies on several assumptions that restrict its domain of validity. In the two last decades, some extensions or alternative models have been developed in order to predict the dynamical response of a multilayer system [8, 9, 10, 13, 14, 15]. The method presented in [10] also consists in applying the usual relations of the SEA for a set of weakly coupled resonators which rely on CPP that can be calculated analytically under some limitations. For example, using such a method with highly dissipative materials is still a challenge. In this paper, we propose a method in order to circumvent this limitation and to be able to deal with highly dissipative materials. This method consists in establishing the expressions of CPP for a set of weakly coupled resonators submitted to external forces modeled as uncorrelated stationary processes with constant spectral power density in each frequency band. We give a numerical example that is two acoustic rooms with a composite panel, which is made up of a highly dissipative acoustic fluid layer sandwiched between two thin elastic layer (metal panel). An analysis of the numerical results is presented with the proposed method, some direct modal solutions and an alternative model of SEA. It is observed that the proposed method can correctly and rapidly enough to predict the sound insulation with a small computing capac-

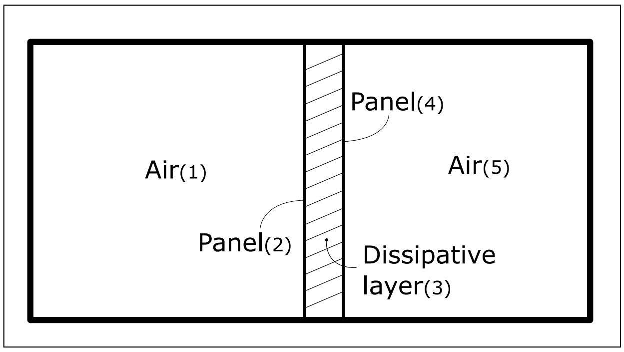

Figure 1. A Fluid-Structure-dissipative fluid-Structure-Fluid system

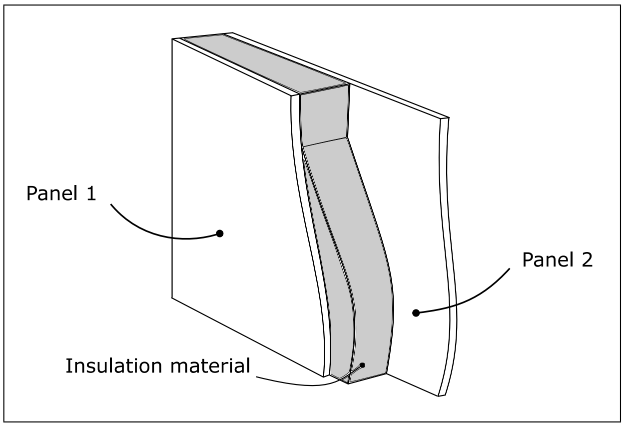

Figure 2. Partition wall with a separative structure

ity requirement for such a system with highly dissipative properties.

2. Description of the dynamical system and computational model

2.1. Dynamical system

For the sake of simplicity, we consider the following dynamical system, which consists in two acoustical cavities that are separated by a composite wall (see Figure 1). The wall is a multilayer viscoelastic medium that is made up of a highly dissipative fluid medium that is sandwiched between two solid elastic layers (see Figure 2).

2.2. Computational model

A computational model is constructed using the finite element method. Hereinafter, the five layers of the dynamical system define five subsystems (i) with $i = 1, \ldots, 5$ that respectively correspond to the first acoustic cavity, the first solid elastic panel layer, the highly dissipative fluid medium, the second elastic panel and the second acoustic cavity. Let $\mathbf{u}^{(1)}$, $\mathbf{u}^{(3)}$ and $\mathbf{u}^{(5)}$ be respectively the vectors of all the nodal values of the velocity potential fields at each node of the finite element mesh of respectively subsystem 1, 3 and 5 and let $\mathbf{u}^{(2)}$ and $\mathbf{u}^{(4)}$ be respectively the vectors of all the nodal values of the displacement field at each node of subsystems 2 and 4. Let $[\mathbb{A}^{(ii)}(\omega)]$ be the dynamical stiffness matrix of subsystem (i). We then have

$$[\mathbb{A}^{(ii)}(\omega)] = -\omega^2[\mathbb{M}^{(ii)}] + i\omega[\mathbb{D}^{(ii)}] + [\mathbb{K}^{(ii)}] \,,$$

where $[\mathbb{M}^{(ii)}]$, $[\mathbb{K}^{(ii)}]$ and $[\mathbb{D}^{(ii)}]$ are respectively the finite element mass, damping and stiffness matrices of subsystem (i). The finite element fluid-solid coupling matrices between subsystems (i) and (j) are denoted as $[\mathbb{C}^{(ij)}]$. Let $\mathbf{f}^{(1)}$ be the finite element vector that corresponds to the linear forms associated with the external loads applied on subsystem (1). We then have,

$$\begin{bmatrix} [\mathbb{A}^{(11)}] & -i\omega[\mathbb{C}^{(12)}] & & & \\ i\omega[\mathbb{C}^{(12)}]^T & [\mathbb{A}^{(22)}] & i\omega[\mathbb{C}^{(23)}] & & \\ & -i\omega[\mathbb{C}^{(23)}]^T & [\mathbb{A}^{(33)}] & -i\omega[\mathbb{C}^{(34)}] & \\ & & i\omega[\mathbb{C}^{(34)}]^T & [\mathbb{A}^{(44)}] & i\omega[\mathbb{C}^{(45)}] \\ & & & -i\omega[\mathbb{C}^{(45)}]^T & [\mathbb{A}^{(55)}] \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \\ \mathbf{u}^{(3)} \\ \mathbf{u}^{(4)} \\ \mathbf{u}^{(5)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{(1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

2.3. Reduced computational model

Let B be a frequency band of analysis included into $\mathbb{R}^+$. We then introduce the rectangular modal matrix $[\Psi_B^{(i)}]$ of subsystem (i) that is such that its α-th column is the eigenvector, which is associated with the α-th smallest angular eigenfrequency belonging to B and to the solutions of the generalized egeinvalue problem $[\mathbb{K}^{(ii)}]\,\mathbf{\Psi}_{\alpha,B}^{(i)} = \lambda_{\alpha,B}^{(i)}\,[\mathbb{M}^{(ii)}]\,\mathbf{\Psi}_{\alpha,B}^{(i)}$. Hence, if $B = [0,\ \omega_{\max}]$ then matrix $[\Psi_B^{(i)}]$ is the usual modal matrix. Let $\mathbf{q}^{(i)}$, for $i = 1, \ldots, 5$, be the vector of the generalized coordinates for the modal decomposition of $\mathbf{u}^{(i)}$ on the frequency band B. We then have $\mathbf{u}^{(i)} = [\Psi_B^{(i)}]\,\mathbf{q}^{(i)}$. Let $[\mathcal{A}_B^{(ii)}(\omega)] = -\omega^2[\mathcal{M}_B^{(ii)}] + i\omega[\mathcal{D}_B^{(ii)}] + [\mathcal{K}_B^{(ii)}]$ be the generalized dynamical stiffness matrix of subsystem (i) where $[\mathcal{M}_B^{(ii)}] = [\Psi_B^{(i)}]^T[\mathbb{M}^{(ii)}][\Psi_B^{(i)}]$, $[\mathcal{D}_B^{(ii)}] = [\Psi_B^{(i)}]^T[\mathbb{D}^{(ii)}][\Psi_B^{(i)}]$ and $[\mathcal{K}_B^{(ii)}] = [\Psi_B^{(i)}]^T[\mathbb{K}^{(ii)}][\Psi_B^{(i)}]$ are respectively the generalized mass, damping and stiffness matrices of subsystem (i). Matrices $[\mathcal{M}_B^{(ii)}]$ and $[\mathcal{K}_B^{(ii)}]$ are definite-positive diagonal matrices and are written as $[\mathcal{M}_B^{(ii)}]_{\alpha\sigma} = \delta_{\alpha\sigma} m_{\alpha,B}^{(i)}$ and $[\mathcal{K}_B^{(ii)}]_{\alpha\sigma} = \delta_{\alpha\sigma} m_{\alpha,B}^{(i)} \lambda_{\alpha,B}^{(i)}$ where $\delta_{\alpha\sigma}$ is equal to 1 if $\alpha = \sigma$ and equal to 0 if $\alpha \neq \sigma$. It is assumed that the generalized damping matrix $[\mathcal{D}_B^{(ii)}]$ is definite-positive diagonal and it is written as $[\mathcal{D}_B^{(ii)}]_{\alpha\sigma} = 2\delta_{\alpha\sigma} m_{\alpha,B}^{(i)} \xi_{\alpha,B}^{(i)} (\lambda_{\alpha,B}^{(i)})^{1/2}$. Let $[C_B^{(ij)}]$ be the generalized fluid-solid coupling matrices between subsystems (i) and (j) defined as $[C_B^{(ij)}] = [\Psi_B^{(i)}]^T\,[\mathbb{C}^{(ij)}]\,[\Psi_B^{(j)}]$. Let $\mathcal{F}^{(1)} = [\Psi_B^{(1)}]^T\,\mathbf{f}^{(1)}$ be the generalized vector of the external forces applied on subsystem (1). We then deduce from Eq. (1),

$$\begin{bmatrix} [\mathcal{A}_B^{(1)}] & -i\omega[C_B^{(12)}] & & & \\ i\omega[C_B^{(12)}]^T & [\mathcal{A}_B^{(2)}] & i\omega[C_B^{(23)}] & & \\ & -i\omega[C_B^{(23)}]^T & [\mathcal{A}_B^{(3)}] & -i\omega[C_B^{(34)}] & \\ & & i\omega[C_B^{(34)}]^T & [\mathcal{A}_B^{(4)}] & i\omega[C_B^{(45)}] \\ & & & -i\omega[C_B^{(45)}]^T & [\mathcal{A}_B^{(5)}] \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{(1)} \\ \mathbf{q}^{(2)} \\ \mathbf{q}^{(3)} \\ \mathbf{q}^{(4)} \\ \mathbf{q}^{(5)} \end{Bmatrix} = \begin{Bmatrix} \mathcal{F}^{(1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (2)$$

The dynamic stiffness matrix in the left hand side of Eq. (2) is denoted as $[\mathcal{A}_B(\omega)]$. Let $[\mathcal{T}_B(\omega)] = [\mathcal{A}_B(\omega)]^{-1}$ be the matrix-valued generalized frequency response function of the whole mechanical system. Matrix $[\mathcal{T}_B(\omega)]$ can be decomposed into 25 blocs denoted as $[\mathcal{T}_B^{(ij)}]$ with $i, j = 1, \ldots, 5$. Let $n_B^{(i)}$ be the truncation order of the modal decomposition of $\mathbf{u}^{(i)}$, and then $n_B^{(i)}$ is the number of columns of modal matrix $[\Psi_B^{(i)}]$. Hence, in all the following sections, for all $1 \leq \alpha \leq n_B^{(i)}$ and $1 \leq \sigma \leq n_B^{(1)}$, $\mathcal{T}_{\alpha\sigma,B}^{(i)}$ denotes the element $[\mathcal{T}_B^{(ij)}]_{\alpha\sigma}$ with $j = 1$.

2.4. Spectral mechanical energy density

The vector of the generalized external forces $\mathcal{F}^{(1)}(\omega)$ is replaced in Eq. (2) by a vector-valued random processes for which the components are modeled as uncorrelated white noises. Consequently, the spectral power density $S_\sigma^{(1)}$ of random process $\mathcal{F}_\sigma^{(1)}$ with $1 \leq \sigma \leq n_B^{(1)}$ is written as $S_\sigma^{(1)}(\omega) = s_\sigma$ where s_σ is a constant and the mechanical energy $E^{(i)}$ of subsystem (i) is modeled as a second-order stationary random process indexed by $\mathbb{R}^+$. Let $e_B^{(i)}$ be the spectral mechanical energy density of subsytem (i) that is defined as, for all $\omega \in \mathbb{R}^+$

$$e_B^{(i)}(\omega) = \sum_{\alpha=1}^{n_B^{(i)}} \sum_{\sigma=1}^{n_B^{(1)}} 2\, s_\sigma m_{\alpha,B}^{(i)} \omega^2 |\mathcal{T}_{\alpha\sigma,B}^{(i)}(\omega)|^2 .$$

If $\langle \cdot \rangle$ denotes the mathematical expectation operator, then $\langle \mathcal{E}^{(i)} \rangle$ is the time-independent mean value of $E^{(i)}$. We then have, for $\omega_{\max}$ large enough and $B_{\max} = [0, \omega_{\max}]$

$$\langle \mathcal{E}^{(i)} \rangle = \int_{B_{\max}} e_{B_{\max}}^{(i)}(\omega)\, d\omega .$$

We introduce $\langle \mathcal{E}_B^{(i)} \rangle$ as the contribution of the frequency band $B \subset B_{\max}$ to the mean mechanical energy $\langle \mathcal{E}^{(i)} \rangle$ such as,

$$\langle \mathcal{E}_B^{(i)} \rangle = \int_B e_{B_{\max}}^{(i)}(\omega)\, d\omega .$$

2.5. Reduced order computational model with non resonant coupling

In [10], authors pointed out that, for such a multilayer mechanical system, the eigenvectors of the fluid cavities are coupled with eigenvectors of the solid elastic layers belonging to a lower frequency band. Consequently, there is an implicit coupling between the acoustic cavities of this mechanical system which is not taken into acount into the constructing of matrix $[\mathcal{A}_B(\omega)]$ and in general

$$\int_B e_{B_{\max}}^{(i)}(\omega)\, d\omega \neq \int_B e_B^{(i)}(\omega)\, d\omega .$$

In order to circumvent this problem, in [10], authors propose to modify matrix $[\mathcal{A}_B(\omega)]$ and add a stiffness coupling between the generalized coordinates of the acoustic cavities between different subsytems (i) and (j), for instance between subsystems (1)-(3) and (3)-(5). The modified generalized dynamical stiffness matrix is then written as

$$[\mathcal{A}_B(\omega)] = \begin{bmatrix} [\mathcal{A}_B^{(1)}] & -i\omega[C_B^{(12)}] & -[\mathcal{K}_B^{(13)}] & & \\ i\omega[C_B^{(12)}]^T & [\mathcal{A}_B^{(2)}] & i\omega[C_B^{(23)}] & & \\ -[\mathcal{K}_B^{(13)}]^T & -i\omega[C_B^{(23)}]^T & [\mathcal{A}_B^{(3)}] & -i\omega[C_B^{(34)}] & -[\mathcal{K}_B^{(35)}] \\ & & i\omega[C_B^{(34)}]^T & [\mathcal{A}_B^{(4)}] & i\omega[C_B^{(45)}] \\ & & -[\mathcal{K}_B^{(35)}]^T & -i\omega[C_B^{(45)}]^T & [\mathcal{A}_B^{(5)}] \end{bmatrix} \quad (3)$$

where the expressions of matrices $[\mathcal{K}_B^{(13)}]$ and $[\mathcal{K}_B^{(35)}]$ are given in[10]. Such a modified matrix $[\mathcal{A}_B(\omega)]$ allows for the following approximation to be deduced from Eq. (3)

$$\langle \mathcal{E}_B^{(i)} \rangle \simeq \int_B e_B^{(i)}(\omega)\, d\omega .$$

2.6. Approximation of the spectral mechanical energy density

In this work, we propose an approximation of $e_B^{(i)}$. Such an approximation can be obtained from the contribution $\langle \mathcal{E}_B^{(i)} \rangle$ with a frequency band B narrow enough such that the variations of $e_B^{(i)}$ over B are slow. For a frequency band $B = [\omega_c - \Delta\omega/2,\ \omega_c + \Delta\omega/2]$ defined by a central angular frequency ω_c and bandwidth $\Delta\omega$, we then have at first order in $\Delta\omega$

$$e_B^{(i)}(\omega_c) \simeq \frac{\langle \mathcal{E}_B^{(i)} \rangle}{\Delta\omega} . \quad (4)$$

3. Energy relations for the approximation of the spectral energy density

In order to compute the approximation introduced in Eq. (4), we have to calculate $\langle \mathcal{E}_B^{(i)} \rangle$ that is the sum of all the mean modal mechanical energy $\langle \mathcal{E}_{\alpha,B}^{(i)} \rangle$. We then have

$$\langle \mathcal{E}_B^{(i)} \rangle = \sum_{\alpha=1}^{n_B^{(i)}} \langle \mathcal{E}_{\alpha,B}^{(i)} \rangle .$$

The proposed method consists in using the principles of the SEA [12] in order to deduce a system of equations for $\langle \mathcal{E}_{\alpha,B}^{(i)} \rangle$. We introduce respectively the contributions $\langle \Pi_{\alpha,B}^{(i)} \rangle$, $\langle \mathcal{P}_{\alpha,B}^{(i)} \rangle$ and $\langle \Pi_{\alpha\sigma,B}^{(ij)} \rangle$ of the frequency band B as respectively the mean modal input power, the mean modal dissipated power and the mean modal exchanged power of subsystems (i) and (j). Following the principles of the SEA for a set of resonators, we then deduce

$$\langle \Pi_{\alpha,B}^{(i)} \rangle = \langle \mathcal{P}_{\alpha,B}^{(i)} \rangle + \sum_{j \neq i}^{5} \sum_{\sigma=1}^{n_B(j)} \langle \Pi_{\alpha\sigma,B}^{(ij)} \rangle , \quad (5)$$

$$\langle \mathcal{P}_{\alpha,B}^{(i)} \rangle = 2\, \xi_{\alpha,B}^{(i)} (\lambda_{\alpha,B}^{(i)})^{1/2} \langle \mathcal{E}_{\alpha,B}^{(i)} \rangle ,$$

and

$$\langle \Pi_{\alpha\sigma,B}^{(ij)} \rangle = \beta_{\alpha\sigma,B}^{(ij)} \left(\langle \mathcal{E}_{\alpha,B}^{(i)} \rangle - \langle \mathcal{E}_{\sigma,B}^{(j)} \rangle \right) , \quad (6)$$

where $\beta_{\alpha\sigma,B}^{(ij)}$ is the modal coupling properties parameters between subsystems (i) and (j). In this work, we introduce the Modal Coupling Power Proportionality (MCPP) that is defined as

$$\beta_{\alpha\sigma,B}^{(ij)} = \frac{X_{\alpha\sigma,B}^{(ij)}}{Y_{\alpha,B}^{(i)}} , \quad (7)$$

where

$$X^{(ij)}_{\alpha\sigma,B} = \int_B i\omega \left\{ (k^{(ij)}_{\alpha\sigma,B} + \omega^2 m^{(ij)}_{\alpha\sigma,B})^2 + \omega^2 (c^{(ij)}_{\alpha\sigma,B})^2 \right\} T^{(j)}_{\sigma,B}(\omega) |T^{(i)}_{\alpha,B}(\omega)|^2 \, d\omega \, , \quad (8)$$

and

$$Y^{(i)}_{\alpha,B} = \int_B m^{(i)}_{\alpha,B} \omega^2 |T^{(i)}_{\alpha,B}(\omega)|^2 \, d\omega \, . \quad (9)$$

in which $k^{(ij)}_{\alpha\sigma,B} = [\mathcal{K}^{(ij)}_B]_{\alpha\sigma}$, $m^{(ij)}_{\alpha\sigma,B} = [\mathcal{M}^{(ij)}_B]_{\alpha\sigma}$ and $c^{(ij)}_{\alpha\sigma,B} = [C^{(ij)}_B]_{\alpha\sigma}$. For the dynamical system presented in section 2.1, we have $m^{(ij)}_{\alpha\sigma,B} = 0$ when $i \neq j$. In addition, in Eq. (8), the generalized frequency response function $T^{(i)}_{\alpha,B}$ associated with the α-th generalized coordinate of subsystem (i) is introduced and is defined as

$$T^{(i)}_{\alpha,B}(\omega) = \left(-\omega^2 m^{(i)}_\alpha + 2\, i\omega\, (\lambda^{(i)}_\alpha)^{1/2} m^{(i)}_\alpha \xi^{(i)}_\alpha + \lambda^{(i)}_\alpha m^{(i)}_\alpha \right)^{-1} \, .$$

In general, it should be noted that $T^{(i)}_{\alpha,B}(\omega) \neq \mathcal{T}^{(i)}_{\alpha\alpha,B}(\omega)$. Moreover, for the special case $B = \mathbb{R}^+$, analytical expression of $\beta^{(ij)}_{\alpha\sigma,\mathbb{R}}$ has been given by many authors [2, 3, 4, 5, 6, 7]. In [10], an approximation $\beta^{(ij)}_{\alpha\sigma,B,\mathrm{SEA}}$ of $\beta^{(ij)}_{\alpha\sigma,B}$ is constructed in replacing the integral over B by an integral over $\mathbb{R}^+$ which allows the regular expressions of SEA to be straightforwardly used. We have

$$\beta^{(ij)}_{\alpha\sigma,B} \simeq \beta^{(ij)}_{\alpha\sigma,B,\mathrm{SEA}} = \frac{X^{(ij)}_{\alpha\sigma,B,\mathrm{SEA}}}{Y^{(i)}_{\alpha,B,\mathrm{SEA}}} \, , \quad (10)$$

where

$$X^{(ij)}_{\alpha\sigma,B,\mathrm{SEA}} = \int_0^{+\infty} i\omega \left\{ (k^{(ij)}_{\alpha\sigma,B} + \omega^2 m^{(ij)}_{\alpha\sigma,B})^2 + \omega^2 (c^{(ij)}_{\alpha\sigma,B})^2 \right\} T^{(j)}_{\sigma,B}(\omega) |T^{(i)}_{\alpha,B}(\omega)|^2 \, d\omega \, , \quad (11)$$

and

$$Y^{(i)}_{\alpha,B,\mathrm{SEA}} = \int_0^{+\infty} m^{(i)}_{\alpha,B} \omega^2 |T^{(i)}_{\alpha,B}(\omega)|^2 \, d\omega \, . \quad (12)$$

Nevertheless, such an approximation is acceptable as long as the generalized damping ratios are weak enough that is to say when the equivalent bandwidth of the dynamical linear filter defined by the frequency response function $T^{(i)}_{\alpha,B}$ are fully embedded into frequency band B. Such a condition is not reached when materials are highly dissipative and it is the reason why we propose a definite integral calculation of the modal coupling power proportionality $\beta^{(ij)}_{\alpha\sigma,B}$. We then deduce an explicit direct solution of Eq. (8) without doing numerical integral in the next section.

4. Direct calculation of the Modal Coupling Power Proportionality (MCPP)

In this section, we are interested in the calculation of $\beta^{(ij)}_{\alpha\sigma,B}$. Rather than doing a numerical integration of Eqs.(8)-(9), we propose a direct calculation. It should be noted that for integral over $\mathbb{R}^+$ and for the case of elastic coupling, such a calculation is already presented in [12]. We then extend it into the case of skew symmetric coupling and for definite integral over any given bounded frequency band $B = [\omega_c - \Delta\omega, \omega_c + \Delta\omega]$. For the sake of simplicity, hereinafter, we present only the obtained relation for the MCPP between subsystems (1) and (2), between which there is only vibroacoustic coupling and no elastic coupling. It can be shown that

$$\beta^{(12)}_{\alpha\sigma,B} = (I_1 J_{22} + I_2 J_{21})/(J_{11} J_{22} - J_{12} J_{21}) \, ,$$

where

$$I_1 = \frac{\Delta_2^2 C^2}{m_1^2 m_2} \int_B \frac{\omega^4}{Q(\omega)} \, d\omega$$

$$I_2 = \frac{\Delta_1^2 C^2}{m_2^2 m_1} \int_B \frac{\omega^4}{Q(\omega)} \, d\omega$$

$$J_{11} = \frac{1}{m_1} \int_B \frac{\omega^6 + (\Delta_2^2 - 2\Lambda_2)\omega^4 + \Lambda_2^2 \omega^2}{Q(\omega)} \, d\omega + \frac{C^2}{2\, m_1^2 m_2} \int_B \frac{-\omega^4 + \Lambda_2 \omega^2}{Q(\omega)} \, d\omega$$

$$J_{22} = \frac{1}{m_2} \int_B \frac{\omega^6 + (\Delta_1^2 - 2\Lambda_1)\omega^4 + \Lambda_1^2 \omega^2}{Q(\omega)} \, d\omega + \frac{C^2}{2\, m_1 m_2^2} \int_B \frac{-\omega^4 + \Lambda_1 \omega^2}{Q(\omega)} \, d\omega$$

$$J_{12} = \frac{C^2}{2\, m_1 m_2^2} \int_B \frac{\omega^4 + \Lambda_1 \omega^2}{Q(\omega)} \, d\omega$$

$$J_{21} = \frac{C^2}{2\, m_1^2 m_2} \int_B \frac{\omega^4 + \Lambda_2 \omega^2}{Q(\omega)} \, d\omega$$

with

$$\Lambda_1 = \lambda^{(1)}_{\alpha,B} \quad \Lambda_2 = \lambda^{(2)}_{\sigma,B}$$

$$\Delta_1 = 2\, \xi^{(1)}_{\alpha,B} \sqrt{\lambda^{(1)}_{\alpha,B}} \quad \Delta_2 = 2\, \xi^{(2)}_{\sigma,B} \sqrt{\lambda^{(2)}_{\sigma,B}}$$

$$m_1 = m^{(1)}_{\alpha,B} \quad m_2 = m^{(2)}_{\sigma,B}$$

$$Q(\omega) = |Q^{(12)}_{\alpha\sigma,B}(\omega)|^2 \quad C = [C^{(12)}_B]_{\alpha\sigma}$$

and

$$Q^{(12)}_{\alpha\sigma,B}(\omega) = \det \begin{pmatrix} [\mathcal{A}^{(1)}_B(\omega)]_{\alpha\alpha} & -i\omega\, [C^{(12)}_B]_{\alpha\sigma} \\ i\omega\, [C^{(12)}_B]_{\alpha\sigma} & [\mathcal{A}^{(2)}_B(\omega)]_{\sigma\sigma} \end{pmatrix}$$

In these integrals, $Q(\omega)$ is an even monic polynomial of degree $q = 8$, with real coefficients and values in $\mathbb{R}^+$. Consequently, we have 8 roots $z_1, \ldots, z_8$ for $Q(\omega)$. We can therefore decompose these integrals with any polynomial numerator $P(\omega)$ of degree $p < q$ into 8 integrals :

$$\int_B \frac{P(\omega)}{Q(\omega)} \, d\omega = \sum_{k=1}^{8} \int_B \frac{R_k(z_k)}{\omega - z_k} \, d\omega$$

with R_k the residues. However, the residues can be analytically calculated when ω is extremely close to the roots. We then have

$$R_k(z_k) = \frac{P(z_k)}{Q'(z_k)}$$

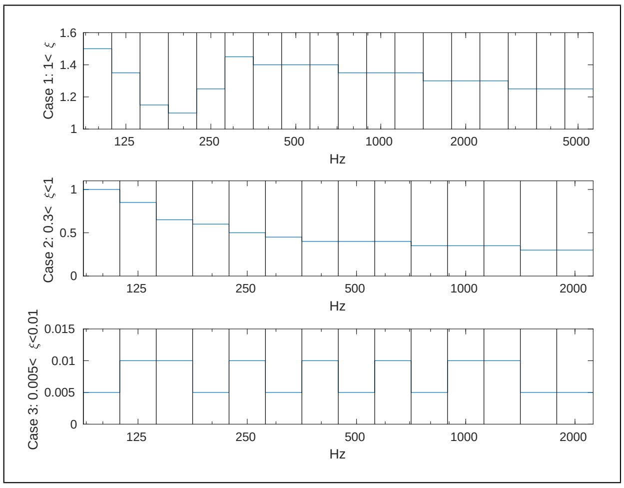

Figure 3. Values of damping ratio in 3 cases

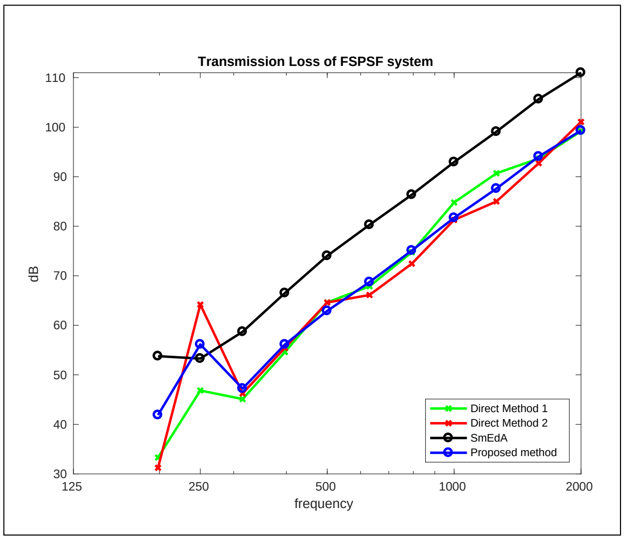

Figure 4. Sound transmission loss estimated with case 1 (damping ratio $1 < \xi^{(3)}$)

in which $Q'(\omega) = dQ(\omega)/d\omega$ and so that

$$\int_B \frac{P(\omega)}{Q(\omega)}\, d\omega = F(\omega_c + \Delta\omega/2) - F(\omega_c - \Delta\omega/2)\,,$$

where

$$F(\omega) = \sum_{k=1}^{8} \frac{P(z_k)}{Q'(z_k)} \ln(\omega - z_k)\,.$$

5. Numerical application

5.1. Description of the dynamical system for the numerical application

This method is compared with a direct calculation of solutions of Eq.(2) (Direct Method 1) and Eq.(3) (Direct Method 2), and with the SmEdA method which consists in using approximation defined by Eqs. (10)-(12). Note that with Direct Method 1, we take $B = [0, \omega_{\max}]$. We choose 1/3 octave as the frequency bandwidth. The properties of each subsystem are listed in the tables I and II, with the length and same width of the whole multilayer system that are equal to $0.8m$ and $0.6m$ respectively. The variation of damping ratio for the dissipative layer is shown in Figure 3 for 3 cases ($1 < \xi^{(3)}, 0.3 < \xi^{(3)} < 1, 0.005 < \xi^{(3)} < 0.001$).

5.2. Energy noise reduction

The quantity of interest for this application is the Sound Transmission Loss (STL) that can directly be quantified by the Energy Noise Reduction (ENR) denoted as $r_{\mathrm{ENR},B}$ and that is defined as the ratio between the mean values of the random total mechanical energy $E_B^{(1)}$ and $E_B^{(5)}$ respectively of subsytems (1) and (5). We then have (for a ENR in dB)

$$r_{\mathrm{ENR},B} = 10 \log_{10}\left(\frac{\langle \mathcal{E}_B^{(1)} \rangle}{\langle \mathcal{E}_B^{(5)} \rangle}\right)\,. \tag{13}$$

5.3. Discussion of the numerical results

After finding all $\beta^{(ij)}_{\alpha\sigma,B}$ matrices, we solve the equations (5)-(6). We then put these $\langle \mathcal{E}_B^{(1)} \rangle$ and $\langle \mathcal{E}_B^{(5)} \rangle$ into equation (13) in order to have a curve of r_{ENR}. From Figures 4-6, it can be observed that the sound transmission loss calculated by the proposed method improves the approximation used by SmEdA (see Eqs. (10) to (12)) for high damping ratio $\xi^{(3)}$. It is encouraging that the results of proposed method match perfectly those of Direct Method 2 in medium-high frequency domain and even in low frequency domain. The results are presented for a frequency band of analysis $[0, \omega_{\max}]$ that is limited to $\omega_{\max}/2\pi = 2245$Hz due to the limitations of the high computational cost induced by Direct Method 1. It can be noted that in very low frequency domain, there are small differences between Direct Method 1 and the proposed method as well as the Direct Method 2. It is due to the errors of approximation from the non-resonant condensation and to the low modal density in each frequency band in very low frequencies that are not sufficient for yielding a good approximation (see Fig 7). However, in medium-and high-frequency domains, the modal density rises very fast, which makes the proposed method and Direct Method 2 accurate enough. Moreover, as the non-resonant condensation is made, the proposed method has smaller matrix size, which reduces its computational cost in medium-high frequency domain.

6. Conclusion

We proposed a method for improving the prediction of the acoustic performances for high dissipative dynamical systems in medium- and high- frequency domains. Since this method carries out an explicit calculation of the Modal Coupling Power Proportionality coefficients of the SEA method, we do not have to inverse the whole dynamical

Table I. Properties of fluid subsystems.

subsystems	thickness (m)	density ρ (kg/m^3)	sound's velocity C (m/s)	damping ratio ξ
(1)	0.8	1.29	340	0.005
(3)	0.1	1.29	340	see Figure 3
(5)	0.7	1.29	340	0.005

Table II. Properties of elastic subsystems.

subsystems	thickness (m)	density ρ (kg/m^3)	Young's Modulus E (Pa)	Poisson's ratio ν	damping ratio ξ
(2)	0.001	7800	2×10^{11}	0.3	0.005
(4)	0.001	7800	2×10^{11}	0.3	0.005

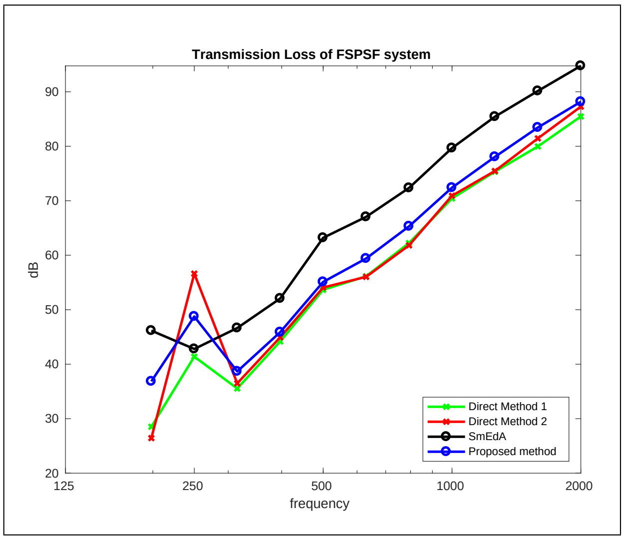

Figure 5. Sound transmission loss estimated with case 2 (damping ratio $0.3 < \xi^{(3)} < 1$)

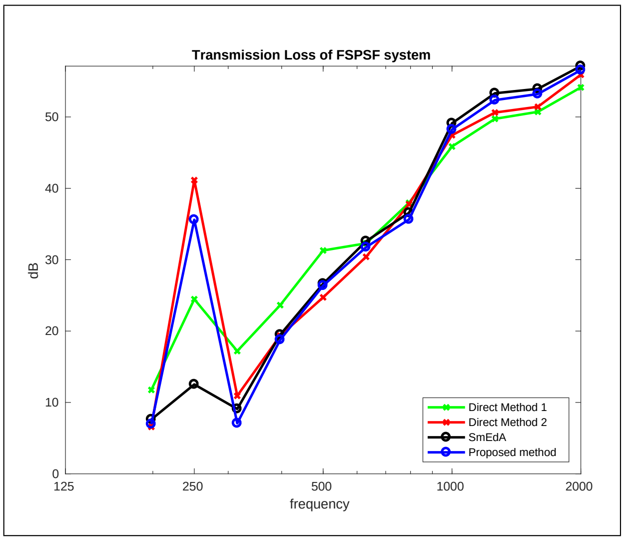

Figure 6. Sound transmission loss estimated with case 3 (damping ratio $0.005 < \xi^{(3)} < 0.01$)

stiffness matrix, which reduces drastically the computational cost when the modal density increases in medium-high frequency domains.

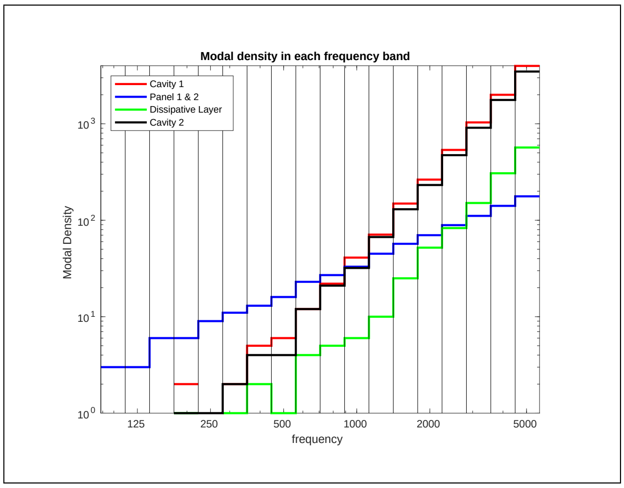

Figure 7. Modal density in each frequency band

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