Guiding acoustic waves over obstacles using linear surface modes

Maaz Farooqui, Yves Aurégan and Vincent Pagneux
Laboratoire d'Acoustique de l'Universite du Maine, UMR CNRS 6613 Av. O Messiaen, 72085 LE MANS Cedex 9, France.

Summary
A reflection-less acoustic cloak inside ducts is proposed in the audible range where plane waves are bended around the object using boundary impedance. The cloak possesses low-reflection and wave-front bending properties, which could approximate the ideal cloak with inhomogeneous and anisotropic distribution of material parameters. The cloak is effective in a broadband frequency range, and the cloaking range is a function of the boundary impedance.

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1. Introduction

Acoustic cloaks has been subject of immense interest in the last decade. Pendry et al proposed an invisible cloak that can perfectly hide arbitrary objects from EM illumination [1]. A similar method is further extended which led to the development of several acoustic cloaks in form of ground [2], carpet [3] cloaks and several others [4, 5] . Chen et al. further confirmed perfect 3D acoustic cloak[6]. In realization of the acoustic cloaks, sonic crystal [6] or acoustic metamaterials [7] can be employed but the local resonance of its inclusions might induce considerable absorption of sound wave [7], which would bring less perfection in the cloak. Hence, zero-dissipation along with the field re-structuring or wavefront bending are of paramount importance in a successful realization of cloaks. All these designs promise potential applications such as sound insulation and invisibility.

Bending of waves using impedance matching concepts have been demonstrated such as an acoustic bend composed of perforated plates [8] or an impedance matching acoustic bend [9]. In these systems, the generation of surface waves on the input and output interfaces of the anisotropic density-near-zero metamaterial induce the sound energy flow to be redistributed and match perfectly with the propagating modes inside the waveguide [9].

In this letter, an acoustic bending/cloaking device is proposed to render the object reflectionless under plane wave propagation, by controlling and guiding the wave path through slow sound phenomenon [10, 11]. A part of sound that is scattered by an impedance wall is confined to a thin layer near the wall, which leads to generation of surface waves [12, 13, 14]. This happens due to the 'pure or spring-like reactance', that influences the airborne particle velocity near the surface and reduces the phase velocity of sound waves in air near the surface. We show that an in-duct purely reacting boundary impedance can act as an effective transformation medium. Full-wave simulations by finite element method (FEM) are performed to demonstrate the low-reflection and wave-front-bending properties of this impedance resulting in the cloaked region in side duct. It has been shown that the cloak exhibits near-ideal performance when it is placed in a duct with plane wave source. It is effective for a broadband frequency range in which the cloaking frequency limit is decided by liner as well as the object dimensions. Our method demonstrates the feasibility of realizing acoustic cloak by ordinary tubes instead of materials with complex structured inclusions. In addition to acoustic cloaking this device renders multi-functional objectives such as paving the way for perfect anechoic termination and several other applications.

Phenomenon description

Acoustic propagation in a 2D duct is considered see Fig. 1. The lower wall is assumed hard and the upper wall is soft which is described by a varying admittance $Y(x)$. The Helmholtz equation, governing the propagation of the acoustic pressure p, is:

$$\Delta p + k^2 p = 0 \quad (1)$$

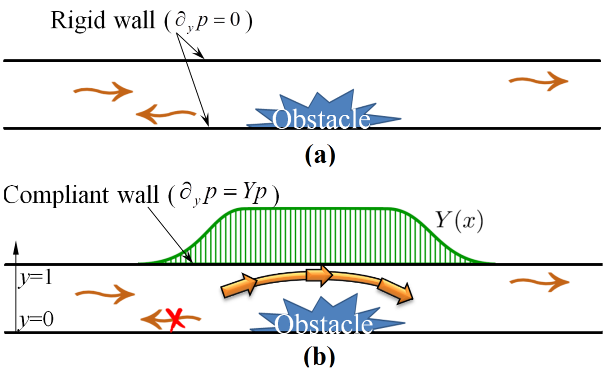

Figure 1. Schematic description of the duct under consideration (a) Uncloaked, (b) Cloaked

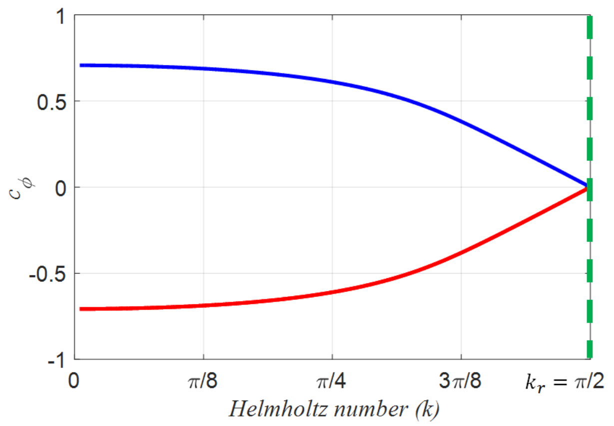

Figure 2. Phase velocity (c_ϕ) variation with Helmholtz number (k), where k_r is a function of length of tube b.

where dimensions are nondimensionalized by the height of the channel H. $k = \omega H/c_0$ is Helmholtz number, ω is the frequency and c_0 is the sound velocity. The boundary conditions are $\partial_y p = 0$ for $y = 0$ which is the hard wall, $\partial_y p = Yp$ for $y = 1$ which is assumed the soft wall. For a uniform admittance, the solution is searched under the form $p = A\cosh(\alpha y)\exp(\mathrm{i}(-\omega t + \beta x))$ where $\alpha^2 = \beta^2 - k^2$ and one obtains the dispersion relation:

$$Y = \alpha\tanh(\alpha) \tag{2}$$

For a rigid wall ($Y = 0$) at low frequencies, only the plane wave can propagate ($\alpha = 0$). If the admittance is positive and, is slowly varying compared to the sound wavelength, α increases progressively as Y increases. It means that the wave is more and more concentrated against the wall. The computations have been made by using FEM based COMSOL where only plane waves are incident from the inlet side of the duct. A silent zone is then created near the wall opposite to the admittance. Any obstacle located in this silent zone will have no influence on wave propagation. This obstacle is invisible in the sense that it does not produce any wave reflection.

Frequency dependence

Among others, a practical realization of the admittance can be done by using small closed tubes of variable lengths perpendicular to the upper wall. Considering lossless tubes, the admittance can be written as:

$$Y(x,k) = k\,\tan(k\,b(x)). \tag{3}$$

To avoid any reflections due to abrupt changes in admittance, a smooth variation of the tube length $b(x)$ is selected:

$$b(x) = \frac{b_0}{2}\left(\tanh\left(\frac{x+l}{d}\right) - \tanh\left(\frac{x-l}{d}\right)\right) \tag{4}$$

where d is a length which characterizes the distance over which the admittance varies, $2l$ is the length between the two smooth variations in the admittance and b_0 is the tube length where the impedance is constant.

At very low frequencies, the admittance goes to zero as $\alpha^2 \simeq k^2 b$ and $\beta = \pm(1+b)^{1/2}k$. The reduced phase velocity $c_\phi = k/\beta$ is then smaller than 1 (the phase velocity is smaller than the sound velocity) [15] as depicted in Fig. 2. When the frequency k goes to the first resonant frequency of the tubes given by $k_r b = \pi/2$, the admittance and α go to ∞ as $\alpha \simeq Y \simeq \pi/(2b(\pi/2 - kb))$. Thus, for frequencies slightly below k_r, the wave decreases exponentially from the wall and has been transformed into a surface wave. Near k_r, the phase velocity of the wave goes linearly to 0 as $c_\phi \simeq (2b/\pi)^2(\pi/2 - kb)$. The propagation is then highly dispersive. Fig. 2, shows c_ϕ plotted as a function of k. It can be visualized from this figure that the phase velocity c_ϕ is always smaller than 1 and tends to zero when k approaches k_r corresponding to the quarter wavelength resonance of the liner.

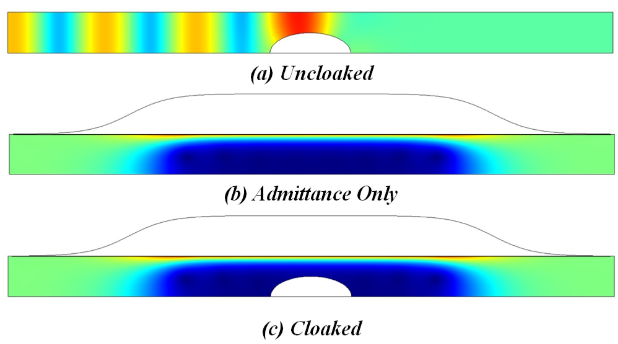

Figure 3. Cloaking an Elliptical obstacle (k=1.38) (a) Absolute pressure field for the obstacle without admittance (Uncloaked), (b) Absolute pressure field for admittance only and (c) Absolute pressure field for obstacle with admittance (Cloaked)

2. Results

2.1. Frequency Domain

In the following results, the maximum tube length has been arbitrarily fixed to $b_0 = 1$ corresponding to height of the channel. The length of the acoustic material is fixed to $l = 6$, height of the obstacle $h_0 = 0.5$, while $d = 2$ which correspond to a admittance variation smooth enough to avoid reflection for frequency larger than $k = \pi/4$. For ideal case of minimal reflections, with assumption of non adiabatic transition($\epsilon << 1$), $R \propto e^{-1/\epsilon}$ which implies $kd \gg 1$. On the example given in Fig. 3, the reflection coefficient of the obstacle (Uncloaked) is $|R| = 0.373$ while when the admittance is present (Cloaked) it is reduced to $|R| = 0.0139$. The shadow zone created by the admittance is impregnable, even with a longer obstacle covering the substantial length of the shadow zone. This feature can be confirmed by comparison of cloaking range depicted in Fig. 4 for admittance and cloaked curves, which remains similar in this range.

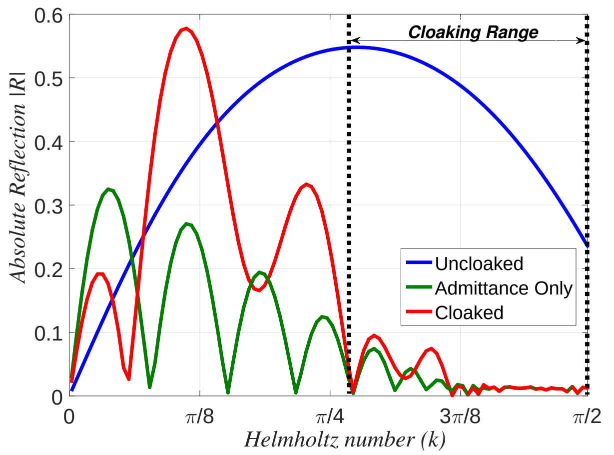

Figure 4. Absolute reflection coefficient $|R|$ for an Elliptical obstacle without admittance: Uncloaked (blue), for admittance without obstacle : Admittance only (green), and for obstacle with admittance: Cloaked (red)

2.2. Time Domain

Acoustic wave propagation in a duct with a full elliptical obstacle is simulated using a Guassian wave packet incident from the left side of the ducts. An implicit temporal discretization procedure is used for this simulation. Fig. 5 depicts the uncloaked case, where the reflection due to the obstacle is obvious as soon as the wave reaches the obstacle ($t \geq 80s$). Fig. 6 depicts the Cloaked case, where there is no reflection even with an enormous obstacle placed inside the duct ($t \geq 80s$). This temporal plots gives a better insight of how the waves bend towards to liner flushed to the upper and lower half of the duct. As explained earlier, the differences in the position

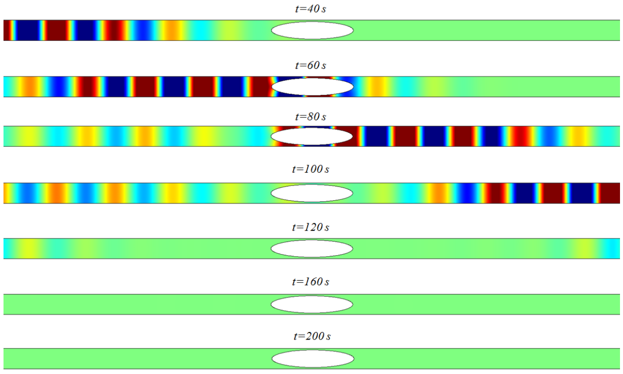

Figure 5. Time-domain results for Pressure field with Gaussian-shaped wave packet on the left side for a duct with only an elliptical obstacle (Uncloaked).

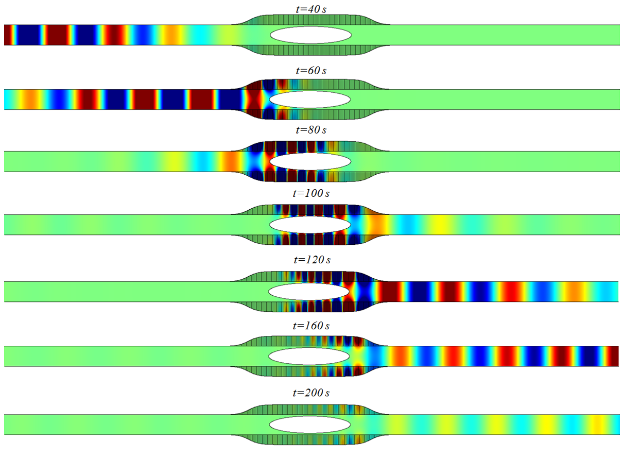

Figure 6. Time-domain results for Pressure field with Gaussian-shaped wave packet on the left side for a duct with an elliptical obstacle and a liner (Cloaked).

of wave-front in Uncloaked (Fig. 5) and Cloaked case (Fig. 6) is attributed to the slow sound phenomenon due to the liner. Due to this slow sound phenomenon, the phase of the acoustic wave in cloaking case will lag behind, the uncloaked case, due to the high dispersion, which is evident in the duct shown in Fig. 6. Again, the length of the acoustic material is fixed to $l = 6$, height of the obstacle $h_0 = 0.98$, and $d = 2$. The gradually slowed sound, reduces the wave celerity and give rise to extreme dispersion as explained earlier. Hence, frequency range of packets was shrunk in order to avoid mutation of the wave packets. Eventually, this increased the length of wave packets.

3. Discussion

In this study, smooth variation of the admittance is adopted. Using the WKB method [14], it can be shown that the absolute value of the reflection is conserved from the obstacle to the rigid wall for this problem. The cloaking range is defined as the frequency range for cloaked case where the absolute reflection is of the order $\leq 20\%$ of uncloaked case. The upper cloaking frequency range is decided by the tube resonance ($Y \to \infty$)), while the lower limit is a function of the cloaking object dimensions as well as the admittance. With the obstacle inside duct, $p(0, y) \propto \cosh(\alpha(y - h_0))$ where h_0 is the height of the obstacle. In order to create a shadow zone encompassing h_0, making use of dispersion relation (Eqn. (2)), we get the first approximation of low frequency limit of the cloak, given by $Y(H - h_0) \gg 1$. It should be noted, that this cloaking frequency range is the minimum possible cloaking range for this elliptical obstacle. Any modifications of the obstacle may further modify the low frequency limit. While the dimensions of shadow zone are proportional to the dimensions of the liner as in Fig. 3(b), the broadband nature of the cloak depends on the liner as well as the geometry of object. For this reason a smooth elliptical obstacle is chosen to aid in achieving the broadband nature. The reflection shown in the green curve Fig. 4, corresponds to the lined duct without any obstacle. It represents merely a smooth duct expansion. When an obstacle is placed, the red curve demonstrates the scattering induced in the system due to the obstacle, but it remains approximately similar to green curve in the cloaking range. This confirms, the phenomena of making that obstacle invisible or more precisely reflection-less.

4. Conclusion

In summary, manipulating/redirecting waves with metamaterials had been a topic of immense interest, and is amplified by the tremendous advancements in cloaks. In this paper, we construct a broadband reflection-less cloak inside ducts with smoothly varying tubes. Full wave simulations confirm that the object inside the cloak can be well hided when the incident plane wave propagates in the duct. Moreover, we numerically demonstrate that this cloak is able to effectively shield the object on the hard plane. The concealed object and the silent zone together can be almost completely shielded to the background medium.

The proposed scheme for designing these in-duct reflection-less cloaks features broadband and bi-directional performance due to symmetry, and allows the parameters of cloaking media to be independent of the cloaked objects i.e. any arbitrary shape can be placed in the silent zone without effecting the cloak efficiency. The simplicity of this cloak lies in the straight-forward designing procedure and the feasibility of realizing it by ordinary tubes instead of acoustic metamaterial with complex structured inclusions. These factors make this cloak emerge as a prominent milestone in this domain of research.

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