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Effect of Mesh Size for Modeling Impulse Responses of Acoustic Spaces via Finite Element Method in the Time Domain

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Summary
The precise calculation of the impulse response of an acoustic space, especially in the low frequency area below the Schroeder frequency, can be achieved with the application of the Finite Element Method in the Time Domain (FEMTD). Mesh size is one of the most important considerations when applying the FEMTD. A rule of thumb for mesh creation is that of λ/h=5 where λ and h respectively denote wavelength of upper limit frequency and the maximum nodal distance.
For this study, calculations of impulse response were performed in virtual 3d spaces with varying reverberation time. Varying mesh sizes were created for each case with maximum nodal distance of the mesh above and below the limit of λ/h=5. Other considerations that were taken into account were the proper selection of source, accurate representation of the impedances of walls, time scales, stepping method and the type of elements. The correlation between the impulse response obtained with the smaller nodal distance size and the impulse responses obtained with higher nodal distance sizes was assessed.
The results indicate that there is a decreasing correlation of the impulse responses over time compared with impulse responses obtained with appropriate mesh size. Also the results suggest that there is an association between appropriate mesh size and reverberation time in acoustic spaces for FEMTD calculations of impulse responses.
Implications of the findings suggest that for impulse response calculation the preferred mesh size should be adaptable to the decay and reverberation time of a space in order certain accuracy to be achieved.

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1. Introduction

An important consideration for Finite Element Method (FEM) and FEMTD is the correct implementation of meshes. Solutions to acoustic problems are wavelike. The waves are characterized by a wavelength λ in space, whose value depends on the frequency and speed of sound c in the medium according to λ = c/f. This wavelength has to be resolved by the mesh. To represent a wave, it is obvious that the mesh elements must be smaller than the wavelength in order the wave to be resolved. That is, there needs to be several degrees of freedom per wavelength in the direction of propagation. The smallest element side length that can be used is determined by the shortest wavelength, that is, highest frequency, to be analysed.

Typically five or six elements are used [1, 2]. Schmiechen [3] states that two points per wavelength are strictly sufficient, but would not lead to accurate mode shapes so a factor of three to five is advised. Wojcik [4] reports computational results with five percent error using nine and two percent error using 18 linear elements per wavelength. Harari [5] proposes 10 nodes per wavelength or more similar to Thompson [6]. Zienkiewicz [7] states that ‘a rule of thumb’ which has been used for some time, is that there should be 10 nodes per wavelength. Marburg found that six elements per wavelength can provide acceptable accuracy [8] similar to Ihlenburg's comprehensive study on finite element error analysis [9]. In a similar fashion Otsuru tested the accuracy for the meshes in the field room acoustics for different elements [10]. He found that the condition $\lambda/h > 4$ assures successful interpolation of peaks in mode shapes and small errors in the eigenfrequency approximation.

Previous work in the field of acoustics has focused on setting mesh restrictions for FEM mainly in the frequency domain. Few researchers have addressed the problem for correct mesh restrictions for FEM in the time domain [11], [12].

In the following years FEMTD for the calculation of impulse responses of acoustic spaces could possibly become widely used in real life applications. The aim of our work is to further extend current knowledge and shed new light on mesh restrictions appropriate for the method.

This study set out to explore the effect of mesh size for FEMTD. Calculations of impulse responses were performed in spaces with varying reverberation time and with varying mesh sizes. Taken together, the results suggest that there is an association between appropriate mesh size and reverberation time in acoustic spaces. The results of this research support the idea that for impulse response calculation via FEMTD the preferred mesh size should be adaptable to the decay and reverberation time of a space in order certain accuracy to be achieved.

Chapter 2 is concerned with the FEM setup employed for this study while chapter 3 presents the findings of the research. Discussion section analyses the data gathered and addresses the research questions in turn. Our conclusions are drawn in the final chapter.

2. FEM setup

The linearized inhomogeneous wave equation is the form of the wave equation that was applied in this study. The finite element formulation is obtained by testing linearized inhomogeneous wave equation (Eq. 1) using the Galerkin method. The finite element formulation of the linearized inhomogeneous wave equation is presented in [13-15]. By applying the FEM in the time domain the calculation of the impulse response of an acoustic space can be derived.

$$\Delta p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{dQ}{\partial t} \tag{1}$$

A Gaussian Pulse point source was used in this study. A Gaussian time profile is defined in terms of its amplitude A, its frequency bandwidth f_0, and the pulse peak time t_p [16, 17]. The spectrum of the pulse is similar to a low pass filter with the frequency bandwidth easily adjusted by controlling the width of the pulse. The governing equations (Eq. 2) were implemented in the Finite Element Analysis.

$$\begin{gathered} Q = -A2\pi^2 f_0^2 \left(t - t_p\right) e^{-\pi^2 f_0^2 \left(t - t_p\right)^2} \\ t_p - \frac{1}{f_0} < t < t_p + \frac{1}{f_0} \end{gathered} \tag{2}$$

In the finite element formulation, the modeling of the walls was carried out using the acoustic impedance. The wall impedance is a quantity which closely emulates the physical behavior of a wall. It is based on the particle velocity normal to the wall which is generated by a given sound pressure at the surface. Considering the incident condition of acoustic wave to the boundary surface, the normalized acoustic impedance of wall surfaces were calculated by substituting the random incidence absorption coefficient of the walls into the Eq. 3 [18, 19].

$$Z = \rho_0 c \frac{1 + \sqrt{1 - \alpha}}{1 - \sqrt{1 - \alpha}} \tag{3}$$

For the application of the FEM in the time domain an appropriate time step was chosen. The time step is dictated by the Courant–Friedrichs–Lewy (CFL) condition [20, 21]. The condition states that the time step must be kept small enough so that information has enough time to propagate through the space discretization. The principle behind the condition is that, for example, if a wave is moving across a discrete spatial grid and its amplitude at discrete time steps of equal duration is to be calculated, then this duration must be less than the time for the wave to travel to adjacent grid points.

For the solution of the wave equation in the time domain with the use of finite elements, a time stepping method is necessary. The Generalized-α [22, 23] time stepping method was applied for this study. Finally the Lagrange 2nd-order tetrahedral elements were used in the finite element formulation. Shape functions can be found in Atalla [24].

3. Results

Figure 1 displays an overview of impulse response calculations for the same 3d space with the application of different meshes. The wall impedances were selected in this case so that the resulting impulse response of the room is would be less than 0.3 sec.

The meshes for the first two impulse responses that are presented in Fig.1 were created with the restriction $\lambda/h<5$. For the third impulse response calculated the restriction was $\lambda/h=5$ and finally in the last case the restriction was $\lambda/h>5$.

The most important observation to emerge from the impulse response comparison in Fig.1 is that in cases 3 and 4, the impulse responses are identical. In the first two cases were the restriction follow $\lambda/h<5$ the impulse responses have differences compared with cases 3 and 4. In Fig.2 and Fig. 3 there is a comparison of the impulse responses for the first two cases in comparison with the impulse response obtained with the mesh restriction $\lambda/h=5$. The results indicate, as expected, that the closer the condition $\lambda/h=5$ is satisfied, the closer the impulse response appears to be with the one calculated with the condition $\lambda/h=5$. Also in Fig.2 and 3 there is a trend of a decreasing correlation of the impulse responses over time.

For the second set of calculations the wall impedances were altered in the same 3d space that was used in the previous calculations, resulting in less absorptive walls and with a room with a longer impulse response. Impulse responses were then calculated for two different mesh restrictions. The new calculations are presented in Figure 4. These results revealed that while the impulse responses in the initial stages are similar, the differences are increasing over time. There seems to be a steady decline of the impulse responses correlation even though the condition $\lambda/h=5$ is met. The next chapter, therefore, moves on to discuss the findings.

4. Discussion

The first question in this study sought to determine if the rule of thumb $\lambda/h=5$ is adequate for mesh creation for the FEMTD. Results presented in Fig. 1 support this hypothesis. However results from the second set of calculations, presented in Fig.4 showed that for longer reverberation time greater number of elements per wavelength is needed in order certain accuracy to be achieved.

A second finding of this study is that the mesh restriction $\lambda/h=5$ will provide good correlation between calculated impulse responses in the early region of the impulse response but with an accuracy that is decreasing over time compared with impulse responses followed the restriction $\lambda/h>5$. The effect probably is going to be greater for acoustic spaces with longer reverberation times.

Our experiments are consistent with previous results where FEMTD was applied for the calculation of the impulse response of an acoustic space [11], [12]. For these studies high values of the cross-correlation coefficient were estimated especially in the early time region of impulse responses calculated for nodal distances following $\lambda/h=5$ compared with the measured impulse responses of the acoustic spaces. However results indicated that there is a decreasing step of the cross-correlation coefficient over time.

A possible explanation for this might be, as Astley states [25] ‘small phase differences between the exact and computed solution may not contribute significantly to numerical error over a single wavelength but accumulate over many wavelengths to give a large global error’. This seems to explain why longer reverberation times provide greater deviation of the impulse responses over time.

This study sets out to extend our knowledge of the correct application of meshes especially for the FEMTD. The results point to the likelihood that in

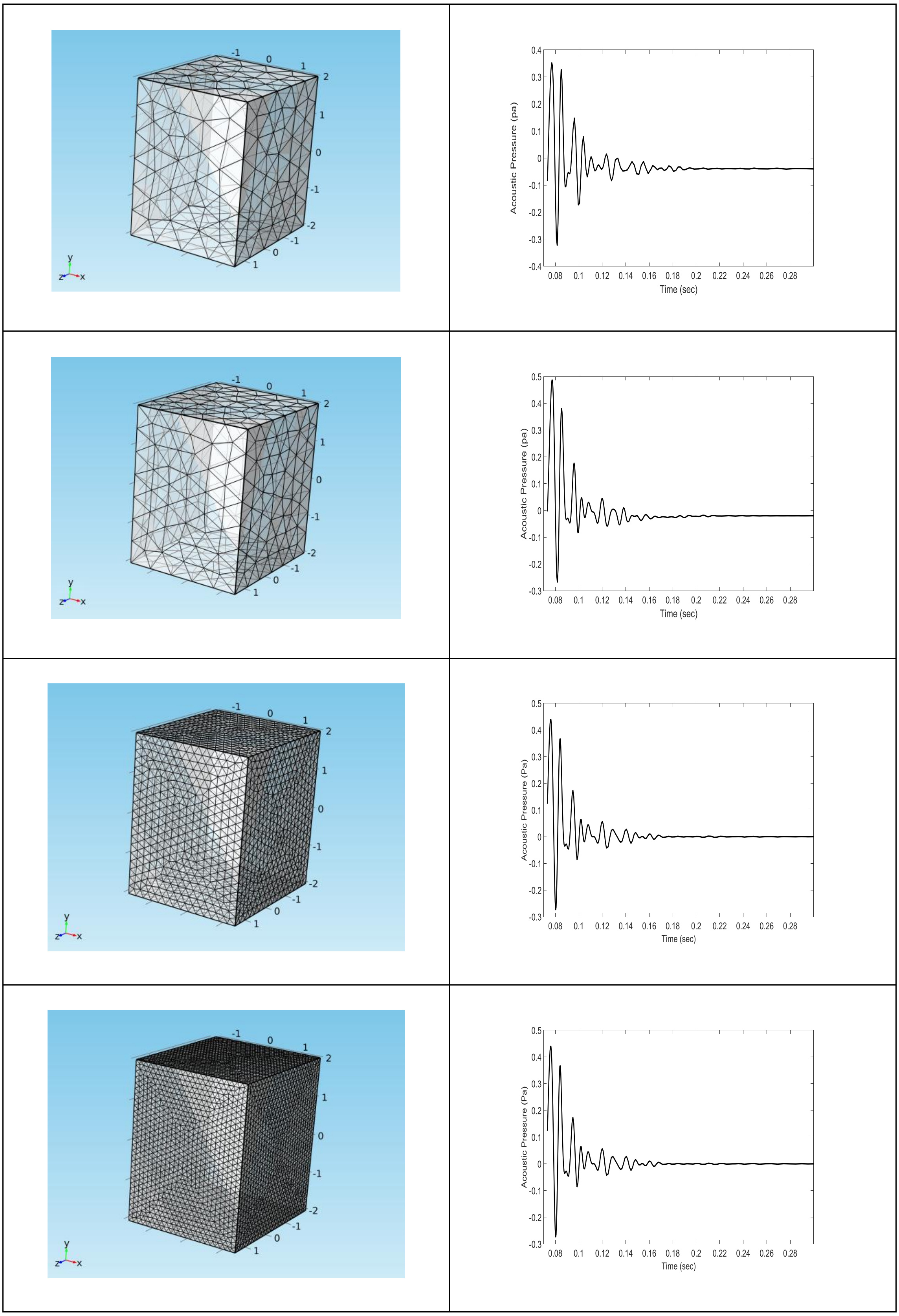

Figure 1 Impulse Response calculations in a 3d space with different meshes (Mesh 1,2: λ/h<5, mesh 3: λ/h=5, mesh 4: λ/h>5)

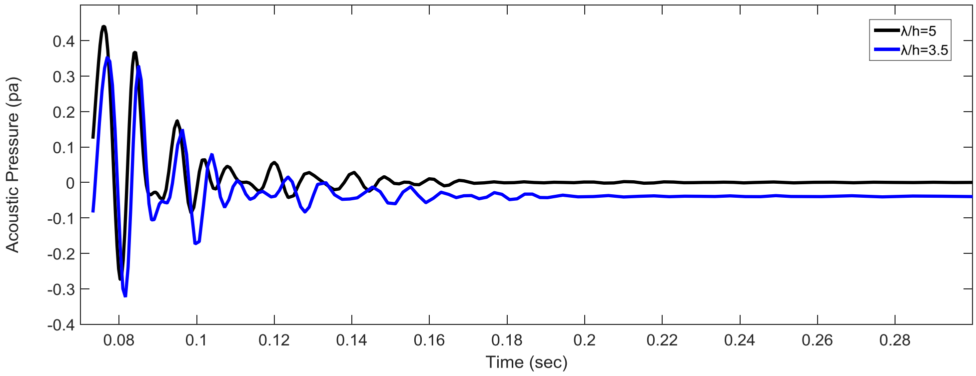

Figure 2 Comparison of the Impulse Responses for different meshes (Blue: Mesh size, λ/h=3.5, Black: Mesh size, λ/h=5)

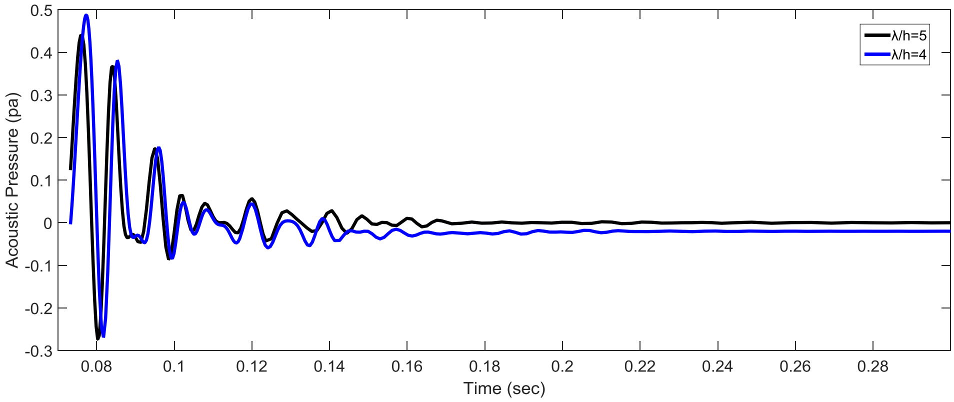

Figure 3 Comparison of the Impulse Responses for different meshes (Blue: Mesh size, λ/h=4, Black: Mesh size, λ/h=5)

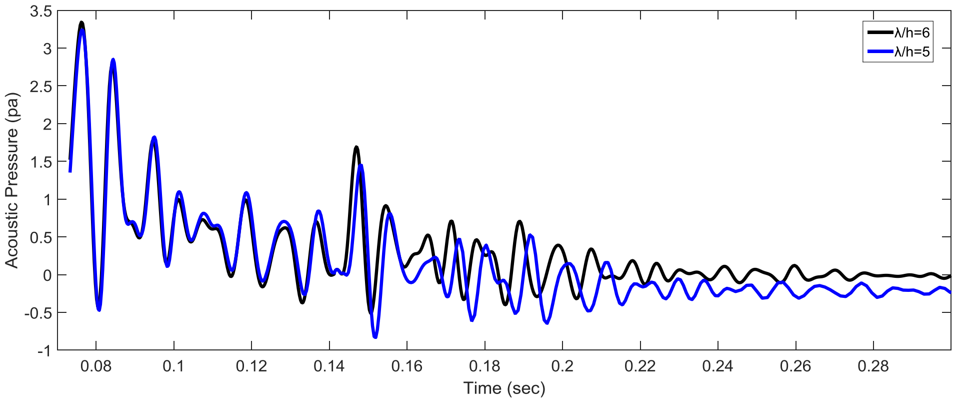

Figure 4 Comparison of the Impulse Responses for different meshes (Blue: Mesh size, λ/h=5, Black: Mesh size, λ/h=6)

order certain accuracy to be achieved, the mesh size should be adaptable to the reverberation time of the space. Hence spaces with greater reverberation time should require a greater number of elements per wavelength in the mesh.

The implications of these findings are significant. Mesh restrictions imposed so far for FEM in the field of acoustics seems to be reasonable and provide good results but mainly for calculation in the frequency domain. For calculations in the time domain it appears that mesh requirements depends on the time duration of the impulse response. Longer impulse responses require smaller element size than the current restrictions proposed for more accurate calculations.

The conclusions of the study should be treated with caution. More research on this topic needs to be undertaken before the association between mesh size and reverberation time is more clearly understood. The present study has only studied a cubic room. To develop a full picture of appropriate mesh restrictions for FEMTD, additional studies will be needed that explore rooms with variant shapes and variant absorptive and diffusive materials on the walls. These topics are reserved for future work.

5. Conclusions

We have performed impulse response calculations in virtual 3d spaces with varying reverberation time with the use of FEMTD. Taken together, the results suggest that there is an association between appropriate mesh size and reverberation time in acoustic spaces.

Implications of the findings suggest that for impulse response calculation via FEMTD the preferred mesh size should be adaptable to the decay and reverberation time of a space in order certain accuracy to be achieved. In the light of these findings, we believe that our analysis may contribute to implementing appropriate mesh restrictions according to reverberation time when implementing FEMTD.

The present study has studied a room with a cubic shape. To develop a full picture of appropriate mesh restrictions for FEMTD, additional studies will be needed. Our investigations into this area are still in progress and seem likely to confirm our hypothesis.

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