Transient response of a multi-storey frame with hysteretic behaviour to white noise excitation

Holger Waubke and Christian H. Kasess
Acoustics Research Institute, Austrian Academy of Sciences, Wohllebengasse 12-14, 1040 Vienna, Austria

Summary

In a recent publication, the stationary response of multi storey frames with hysteretic springs representing the massless columns and rigid masses for the floors is presented. As an extension, the transient behavior without an elastic restoring force shall be investigated. For the one degree of freedom case the results using the Gaussian closure technique are equivalent to the results of the statistical linearization. However, this is not the case for a multi degree of freedom case. Additionally, the Gaussian closure technique allows for a system without a linear restoring force. In this case, no stationarity will be reached and therefore only a transient analysis of the displacements of the floors is possible. The hysteresis is simulated using Bouc's original model. The response is calculated with an explicit first order time step procedure for the moments derived from Gaussian closure technique. The derivation of the solution is done analytically using a recursion algorithm. This solution is based on relative displacements that can be derived in a simple manner for chain like structures like multi-storey frames in 2D. Therefore, the results produced are the relative displacements that can be transformed to absolute displacement, because the Gaussian closure will calculate all variances and co-variances of the system. The major disadvantage of the Gaussian closure method is that all co-variances and mean values have to be determined, because the number of equations that have to be solved increases almost quadratically with the number of mechanical degrees of freedom. The results of the Gaussian closure technique are compared to the results from the Monte-Carlo method.

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1. Introduction

The Bouc model [1, 2] is a relatively simple model to describe the elastic-plastic behavior of structural components. A mathematical description of a dynamic multi degree of freedom (MDOF) system with the hysteretic behavior described by Bouc is possible using a state vector description, which is particularly useful in the stochastic case [3]. The state vector description depends only on first order differentials. The usual degrees of freedom are the displacements of the nodes. Introducing the velocity of the degrees of freedom (DOF) as an additional state vector variable a description of a dynamical system is possible. The Bouc model adds a third type of variables to the state vector the force proportional displacements of the hysteretic elements.
If the loading of the system is of random type, the Monte-Carlo method allows estimating the exact solution of the dynamical system. However, this method is very time consuming, if appropriate estimates are desired. If the loading is a white noise random process the Kolmogorov [4] equations allows deriving differential equations depending on the multi-variate conditional probability density of the random response. However, no analytic solution for this complex differential equation exist. Furthermore, although the moment equations are exact, but in most cases, also for the Bouc model the number of unknown moments is always higher, than the given number of equations for their solution. Therefore, an assumption on the higher moments is needed.
In the case of the Gaussian closure [5,6] only the first and second order moments are derived by the nonlinear differential equations and the higher order moments are converted into sequences of first and second order moments using Isserlis theorem [7,8].

Additionally, an approximation by the Gaussian closure, assuming that the multi-variate probability density is of Gaussian type, allows deriving an analytic solution for the differential equations depending on the moments of the probability density.
The main difficulty is the solution of the integrals in the Bouc model that depend on sign-functions. The sign functions have the state vector variables as an argument. For a single degree of freedom system the argument are the velocities and the force proportional displacement of the DOF. For a MDOF system, the difficulty occurs that the argument becomes the difference of the values at two nodes. Therefore, a description of the system in relative coordinates is needed. System with only one direction of freedom e.q. multi-storey frames allow for a simple representation as a chain like system that can be simply described in relative coordinates only.

2. Simple liquid tank model

Similar to multi-storey frames, a simple model for a fluid filled tank can also be described in relative coordinates, which shall be investigated in more detail here. In this model, the tank with its mass is one degree of freedom and the swapping mass and impulsive mass of the fluid are two additional degrees of freedom.
A ground motion presenting an earthquake event gives the excitation. Clearly, a white noise excitation is not a valid assumption for an earthquake event. Therefore, the white noise excitation is filtered by a linear differential equation of the second order. A filtering by a third order differential equation proposed in some literature for special ground profiles is also possible. The DOFs of the filter are simply added to the state vector of the system.
The earthquake events are often assumed to behave stationary. However, this is not the case in practice. Therefore, a time dependent intensity of the random process will be added to the model. In a future, a change of the filter parameters will be used to be able to change also the frequency content of the random excitation with respect to time. To calculate the transient response of the system, an explicitly given first order differential equation with respect to time for the moments has to be solved. This done by a simple time step procedure.

3. Numerical model

The two special integrals that have to be solved consist of a polynomial of the sate vector variables, a sign function with the relative velocity or the force proportional displacement of the related DOF and the probability density. If the variable given in the sign function is used as the last integration variable the multi-dimensional integral depending on all variables in the state vector can be solved analytically using symbolic algebra (MAPLE). For the single degree of freedom (SDOF) case, the multi-dimensional integral can be solved analytically in one-step [9,10]. For the MDOF case [11] the number of state vector variables changes from model to the other. Therefore, a sequence of equations giving the analytical solution for the integration with respect to one state vector variable needs to be derived. The integration with respect to the variable occurring in the sign function leads to a solution that depend on the Gaussian distribution itself. This function does not allow further integrations and is therefore the last step.

The Gaussian closure for a SDOF model behaves very stable. Here the integration is done in a single step and delta-distributions caused by variances with the value zero are not dangerous. In the MDOF case, the integration is done separately for every state vector variable and divisions by the variances occur in the equations. Therefore, the variances are limited to values above 10^{-8}. Three effects lead in some time steps to zero or negative values for the variances:

- The explicit time step procedure produces some artefacts caused by the extrapolation,
- the time varying intensity of the random process leads to nearly deterministic behavior after the excitation has ended,
- the ground vibration alone leads highly correlated behavior of the upper parts of the model.

The last argument also leads to correction coefficients about or above one, or minus one that also lead to a breakdown of the time step iteration. This occurs especially between tank and impulsive mass, because these two DOFs are coupled very stiffly with a high resonance frequency and the excitation is limited to low frequencies.
To correct those values the variances are limited to small positive values and the correlation coefficients are limited to an interval [-0.999999, 0.999999].

With these measures, the iteration is stable and useful results are produced compared to the Monte-Carlo method, but the performance is drastically reduced. For a SDOF case, a performance factor of 10000 for the Gaussian closure was found. For the MDOF case, the performance factor compared to Monte-Carlo method is only 10.
To increase the performance is an ongoing work.
The Monte-Carlo method is also implemented as an explicit time step procedure based on the state vector variables. For a good estimation, 100.000 realizations were used. The time step was of the same size as for the Gaussian closure. Usually the Gaussian closure can use much larger time steps, because the moments iterated in the Gaussian closure behave much smoother than the realizations used in the Monte-Carlo method. This could further increase the performance considerably. However, it has to tested, whether a larger time step has an influence of stability of the iteration. Additionally, the transient load hinders the approaching of stationary values for the moments therefore, the time steps cannot be increased with respect to time in a manner as it was used for a stationary load before.

4. Results

In the numerical tests, dimensionless coordinates are used to generalize the results. For a first test run the results from 0 to 100 time units is presented in Figure 1. The Gaussian closure slightly underestimates the results from the Monte-Carlo simulation.
The intensity of the load is given with 0.5. The time window is a tapered window with transition regions from 0 to 1 in the time interval [0,1] and a transition region from 1 to 0 in the time interval [30,50]. The function in the transition regions is of cosine type.
The parameter of the load and the filter are

Mass	0.06667,
Damping	0.3,
Stiffness	1.06667.

The parameters used for the tank coupled with the ground are

Mass	1.0,
Damping	0.0,
Stiffness	1.0,
Hysteretic parameter A	1.0,
Hysteretic parameter γ	0.5,
Hysteretic parameter θ	0.5.

A pre-stressing is added to the hysteretic spring with a value of 0.5. A linear elastic part is not added to the hysteretic spring coupling the tank with the ground.
The parameters used for the impulsive mass coupled with the tank are

Mass	0.1,
Damping	0.0,
Stiffness	0.5.

The parameters used for the sloshing mass coupled with the tank are

Mass	0.5,
Damping	0.0,
Stiffness	0.1.

This assumption does not work for the statistical linearization, increases the nonlinearity in the model, and makes the iteration less stable. In addition, the assumption that the damping coefficients of the springs are zero reduces the stability of the iterative method.

5. Conclusion

The Gaussian closure is a method for a fast approximation of MDOF system with elastic-plastic spring elements described by the Bouc model. However, the stability of the iterations needed for the transient case needs some corrections that reduce the performance of the proposed method. The optimization of the iteration with respect to the chosen time step is an ongoing project.
The results of the Gaussian closure fit well to the results of the Monte-Carlo method. It has to be kept in mind that the Gaussian closure is a second order statistical method, therefore the deviations become larger, if the non-linearity of the hysteretic elements increases.

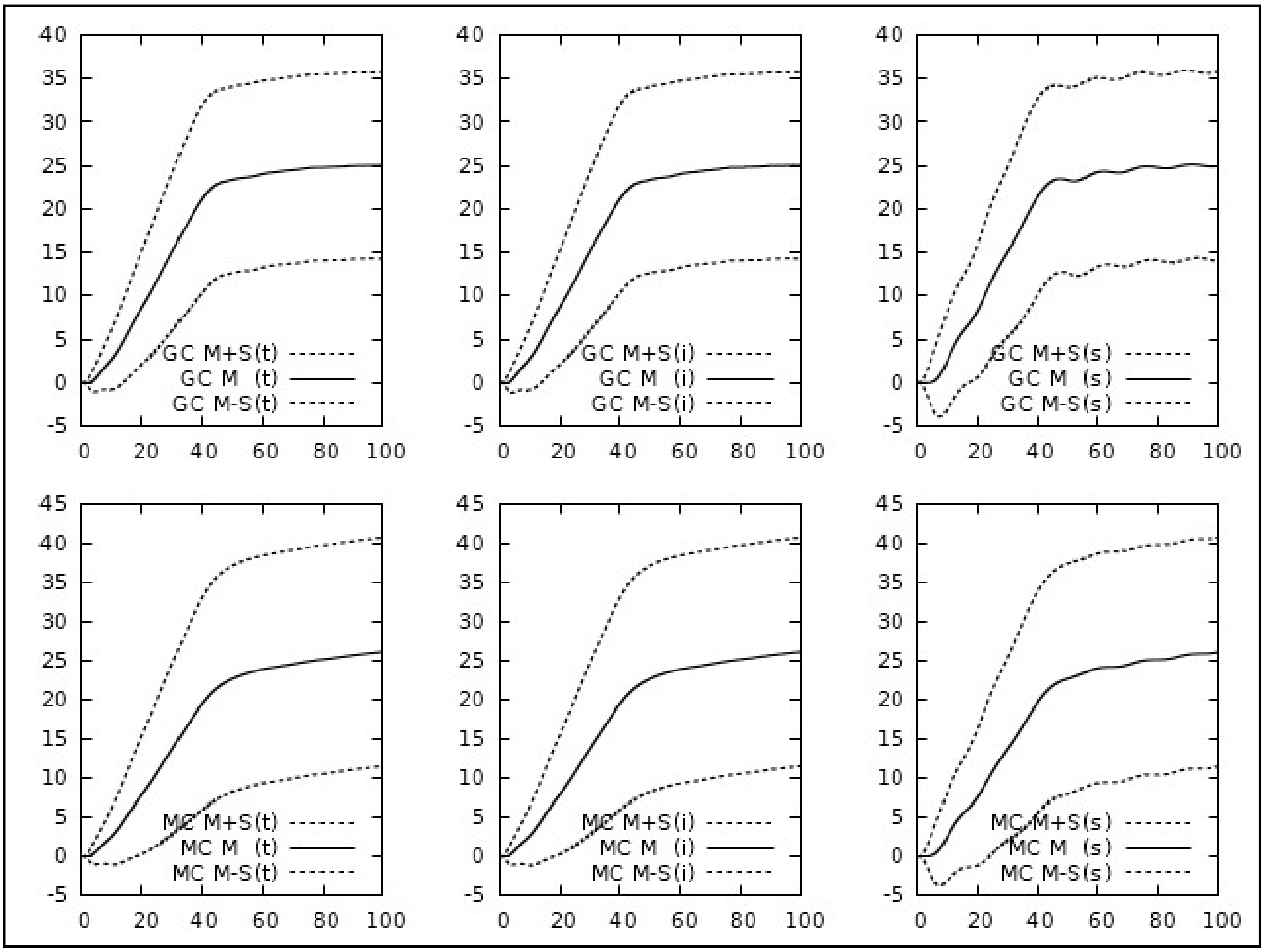

Figure 1: Mean value (M) and standard deviation (S) of Gaussian closure (GC) and Monte-Carlo (MC) method for the tank mass (t), impulsive mass (i) and sloshing mass (s).

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