Redirection of flexural waves in platonic crystal slabs

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Summary
An incoming wave can be redirected into the perpendicular directions in periodic systems when its wavelength is similar to the lattice period. This wave motion property was termed as Poisson-like effect in analogy to the phenomenon in which the deformation of solid materials couples in two perpendicular directions. For flexural waves propagating in a thin plate, the effect of 90° redirection of elastic energy is observed in platonic crystal slabs consisting of periodically perforated free holes. It is demonstrated that the coupling of leaky guided modes with the propagating waves contributes to the anomalous wave motion. For smaller holes, Poisson-like wave motion occurs in company with the Wood anomaly, featured as total reflection due to the excitation of stationary waves propagating along the slab. With the increasing of the size of the holes, the eigenmode gradually evolves to a mixed guided mode featured as waves propagating in two orthogonal directions. In this case, however, the Poisson-like effect is excited when the incoming wave is totally transmitted. This work investigates the evolutionary process of the leaky guided modes and may find an application in energy redirection and waveguiding of flexural waves in platonic crystals.

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1. Introduction

Sonic/phononic crystals are artificially structured composite materials consisting of periodic distributions of scatterers embedded in a fluid/solid matrix [1]. The choice of scatterers enables us to manipulate the dispersive properties of acoustic and elastic waves and, consequently, provides many novel applications [2]. For the case of vibrations in thin plates, the name of platonic crystals is widely used to describe the periodic structures allowing the manipulation of flexural waves [3, 4].

Most of the works focused their attention on strong scatterers, because the high contrast between the matrix and scatterers contributes to the formation of bandgaps. Also, it has been shown that bandgaps at extremely low frequencies are possible by using resonators as building blocks [5]. These new type of structures are called metamaterials and their study is deserving increasing interest and nowadays has become a hot research topic [1, 6].

Nevertheless, it must be emphasized that periodic systems made of weak scatterers are also fascinating because their collective scattering behavior may lead to new physical phenomena [7, 8, 9, 10]. For example, García-Chocano and Sánchez-Dehesa [7] observed an anomalous sound absorption in lattices of cylindrical perforated shells, which are supposed to be acoustically transparent because of their large perforation ratios. This phenomenon was explained in terms of Wood anomaly [11] because the energy transferred to a leaky guided mode travels along the slab, which greatly increases the propagation path of the

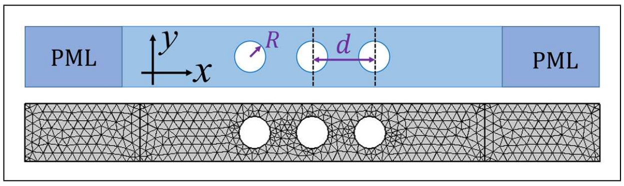

Figure 1. Numerical model of an infinite platonic crystal slab consisting of three layers of free holes.

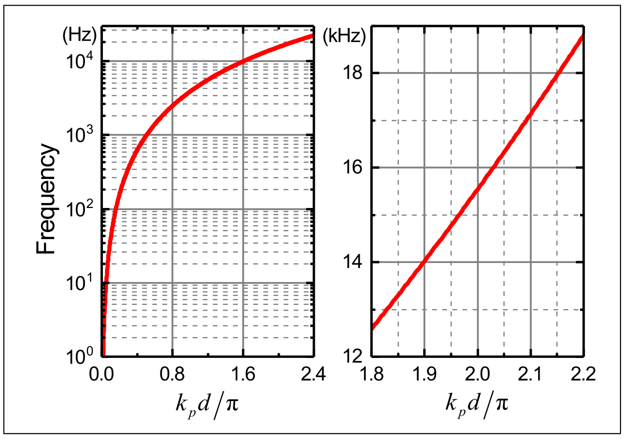

Figure 2. The correspondence of frequencies between reduced units and absolute units for the parameters studied.

sound waves. Two years later, Bozhko *et al.* [8] developed the theory explaining that the effect is also valid for a periodic chain of perforated cylindrical shells. Through the analysis of dispersion relations, they concluded that splitting of sound waves is possible at close frequencies, since both symmetric and anti-symmetric eigenmodes can be excited at oblique incidence. In addition, redirection of acoustic energy by 90° is also achieved by matching Bragg scattering with a quadrupole resonance of elastic shells [9]. The quadrupole resonance couples the normal unidirectional wave motion to the perpendicular one, therefore this 90° redirection is termed as Poisson-like effect in analogy to the Poisson effect in solids.

This work investigates the redirection of flexural waves propagating in periodically perforated thin plates. Compared with other periodic systems, the platonic crystals consisting of perforated holes have an unique advantage in the modulation of scattering property by merely changing the radius of the free holes. As it is shown later, the leaky guided mode of lowest order evolves with increasing radius, nevertheless, it can always be excited to redirect flexural wave to the perpendicular directions. It is just that the phenomenon changes from Wood anomaly to total transmission.

2. Numerical model

Let us consider a platonic crystal slab, structured in a homogeneous aluminum plate, consisting of three layers of free (not clamped) holes in the x-direction and being infinitely long in the y-direction. All the holes are arranged in square lattice with lattice constant $d = 25\,\text{mm}$. The slab functions as a leaky waveguide that guides waves propagating along it. To get a physical insight, we first studied the dispersion relations of the infinite system by employing the commercial finite element package COMSOL. The scheme of the model and the mesh are shown in Figure 1 where a row of three holes is employed as the unit cell. Periodic boundary conditions are considered along both directions in the y-axis. In the horizontal x-axis, a pair of perfectly matched layers (PML) are applied at the borders of the calculation region in order to simulate an infinite plate.

The parameters employed in the calculations are: Young's modulus $E = 69\,\text{GPa}$, Poisson's ratio $\nu = 0.33$, mass density $\rho = 2.7\times10^3\,\text{kg/m}^3$, plate thickness $h = 1\,\text{mm}$ and radius of the holes $R = 3\,\text{mm}$. In what follows, the reduced frequency, $k_p d/\pi$, is adopted because it provides useful information about wavelength λ for the dispersive flexural waves. Here $k_p = 2\pi/\lambda$ is the wave number in the homogeneous plate. The correspondence between reduced units and absolute units, for the parameters studied, is shown in Figure 2. The diffraction limit is defined when the wavelength is equal to the lattice constant, i.e., $k_p d/\pi = 2$, corresponding to 15.54 kHz in this study.

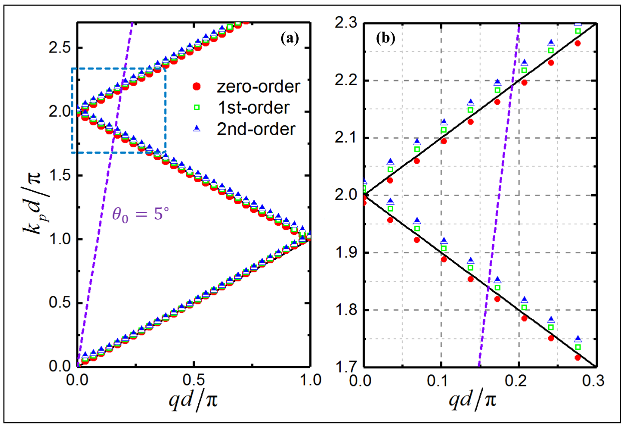

Figure 3. Band structures of an infinite slab containing three layers of free holes. The solid line represents the dispersion relations of a homogeneous plate and other symbols are the real parts of the first three orders of eigenfrequencies calculated with the finite element method. The crossings of the dispersion relations with the dashed line define the frequencies of resonant coupling between the incoming wave and the leaky modes. The right panel provides a zoom view near the diffraction limit, where the wavelength is comparable with the lattice constant.

3. Results and discussion

Figure 3 shows the band structures of an infinite slab containing three layers of free holes. The circles,

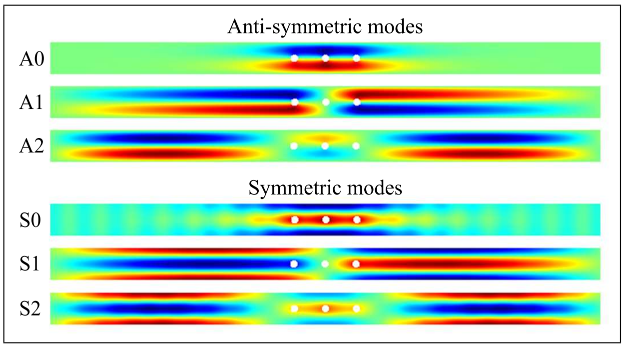

Figure 4. The first three orders of eigenmodes associated with the edges of the narrow gap at the Γ point $qd/\pi = 0$. The symmetric and anti-symmetric modes are categorized according to their symmetry with respect to the x-axis. From top to bottom, the eigenfrequencies (in Hz) are 15125 (A0), 15414+0.5i (A1), 15472+3.1i (A2), 15257+13.2i (S0), 15441+1.9i (S1) and 15478+6.0i (S2), respectively. Note that the eigenfrequencies slightly increase with the order of the guided modes.

squares and triangles represent the real parts of the first three orders of eigenfrequencies calculated within the framework of the finite element method. As shall be shown later, these results are extracted according to the symmetry of the eigenmodes. The solid lines represent the dispersion relations in a homogeneous plate. Free holes with small radius are weak scatterers and, consequently, the bands of the platonic crystals based on them nearly overlap with those of flexural waves propagating in a homogeneous plate. Nevertheless, the bands exhibit extremely narrow gaps near the diffraction limit, as it is observed in the zoom view shown in Fig. 3(b). This little level repulsion is a consequence of the scattering with these relatively small drilled holes.

For a better understanding, the first three orders of eigenmodes at the edges of the narrow gap are depicted in Figure 4. These guided modes are classified into two categories according to the symmetry with respect to the x-axis (see Figure 1). The symmetric and anti-symmetric modes are associated with the upper and lower bands, respectively.

In the case of normal incidence, only the symmetric modes can be excited because they consist of two identical modes traveling along the y-axis but in opposite directions. At slightly oblique incidence, however, both modes can be excited provided the parallel component of the incoming wave vector matches with the Bloch vector of the eigenmodes [8]. To prove this, Figure 5 shows the calculated transmittance when a plane wave impinges on an infinite slab with incident angles $\theta_0 = 0°$ (red solid line) and $\theta_0 = 5°$ (blue dotted line). For the normal incidence ($\theta_0 = 0°$), only one dip is observed near the diffraction limit. It corresponds to the well-known Wood anomaly indicating the excitation of the lowest order symmetric leaky guided mode. The coupling of the incoming waves with the leaky mode is clearly seen in the S0 mode (see Figure 4). Two dips are observed in the spectrum at oblique incidence with $\theta_0 = 5°$, which is attributed to the excitation of the symmetric and anti-symmetric resonant modes. The positions of the resonant dips agree well with the eigenfrequencies predicted by the matching condition, see the crossings of the dispersion relations with the dashed line shown in Figure 3. Notice that only the lowest order guided modes are excited in both cases. This effect can be explained by the eigenfrequencies given in the caption of Figure 4. Their imaginary parts are related with the lifetime of the resonant modes, and larger value means stronger resonance coupling [7]. The imaginary parts of higher order modes are negligible and, therefore, they cannot be excited by the incoming flexural waves.

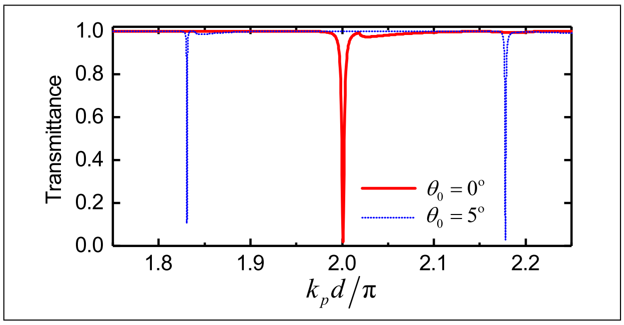

Figure 5. Transmittance spectra obtained when a plane wave impinges on an infinite platonic crystal slab consisting of three layers of free holes. The solid and dotted lines represent the results in cases $\theta_0 = 0°$ and $\theta_0 = 5°$, respectively. From left to right, the frequencies of the three minimums are 1.83, 1.99 and 2.18 (in reduced units), respectively.

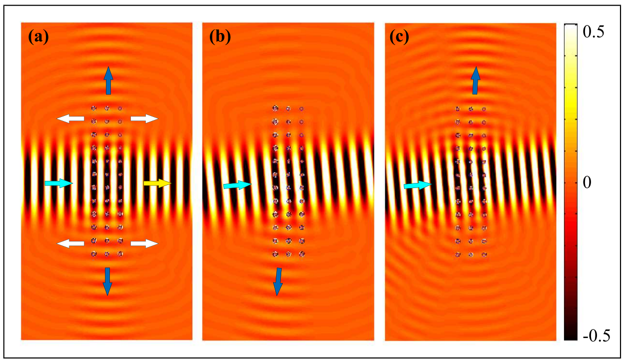

Figure 6. Multiple scattering simulations showing the interaction of an Gaussian beam impinging on a platonic crystal slab. From left to right, the impinging frequencies are 15.40 (a), 13.02 (b) and 18.43 kHz (c), corresponding to the frequencies of the three minimums observed in Figure 5. The arrows indicate the main energy fluxes.

Once the lowest order guided modes are excited, the incoming waves can be redirected to the vertical directions. To prove this, we perform a series of multiple scattering simulations [4] for a finite slab consisting of 3×12 circular holes. The out-of-plane displacement snapshots are displayed in Figure 6 where an Gaussian beam is incoming from the left side. At normal incidence, part of the incoming wave is equally transferred to the vertical sides. During this process, some energy leaks from the slab to the free space, as indicated by the white arrows in Figure 6(a). At oblique incidence, $\theta_0 = 5°$, the symmetry of the incoming wave is broken so that the energy is only redirected to a single side. On the lower band, the group velocity is opposite to the Bloch vector so that the incoming wave is scattered to the abnormal direction, the bottom end of the slab [see Figure 6(b)]. As demonstrated in Ref. [8], this property can be used to split dual-frequency signal for flexural waves.

Now, let us consider the case of strong scatterers, which are achievable just by increasing the radius of the holes. Figure 7(a) depicts three transmittance spectra, corresponding to slabs made of holes with three different radii: 3 mm (solid line), 5 mm (dashed line) and 7 mm (dotted line). For the larger holes, the Wood anomaly disappeared and the extremely narrow deep has developed into a broad transmittance peak. Nevertheless, the underlying mechanism is the same because their appearance is a result of the resonance coupling of the incoming wave with the lowest order leaky guided modes. Figures 7(b) and 7(c) provide the A0 and S0 modes at the Γ point $qd/\pi = 0$ for the case $R = 7\,\mathrm{mm}$. Clearly, the A0 mode shows similar features as its counterpart shown in Figure 4, while for the S0 mode their is a little difference. Figure 7(c) shows a feature of stationary wave containing standing wave modes in both x- and y-directions. Compared with smaller holes, the strong scattering caused by larger holes leads to a mixed mode having eigenfrequency of larger imaginary part, corresponding to 13547+1418.0i Hz for the guided mode shown in Figure 7(c). Therefore, the Poisson-like wave motion is much stronger than that associated with the Wood anomaly, which is demonstrated by comparing the wave patterns in Figures 6(a) and 7(d).

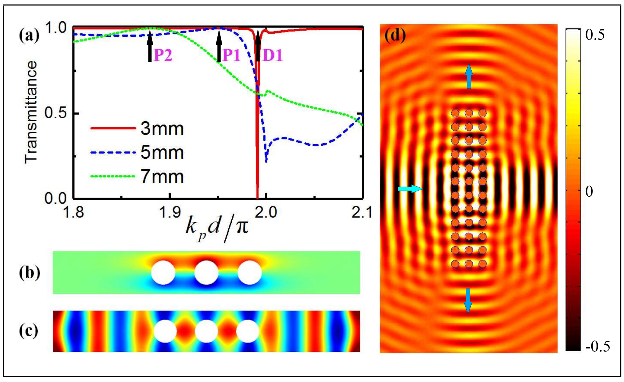

Figure 7. The influence of the radius of the holes on the leaky guided modes. (a) Transmittance spectra for an infinite slab containing holes with radius 3 mm (solid line), 5 mm (dashed line) and 7 mm (dotted line). (b) and (c) depict the patterns of the lowest order guided modes, A0 and S0, respectively. (d) Snapshot of the out-of-plane displacement field describing the Poisson-like effect for flexural waves. It is obtained with an incident frequency of 13.75 kHz, corresponding to 1.88 in reduced units, for the largest holes studied here.

4. Conclusions

In conclusion, we have studied the leaky guided modes of a platonic crystal slab containing three layers of free holes. It is demonstrated that the scattering property determines the S0 mode so that the Wood anomaly gradually evolves to a broad transmittance peak with the increasing of the radius of the holes. Compared with smaller holes, the mixed mode caused by the larger holes can be easily excited by the incoming waves. This effect has potential applications in designing vibration devices for the redirection of flexural waves.

Acknowledgement

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