Observation of Nonlinear Vibro-Acoustic phenomena in the Presence of Elastic Membrane with Different Boundary Conditions

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Summary

Linear behaviors are typically encountered in duct acoustics where sound waves propagate in rigid ducts. In this experimental study, a circular visco-elastic membrane is used as an essentially nonlinear attachment to generate local nonlinearities for a primary linear system constituted by a straight rigid duct. In specific acoustic conditions, the membrane behaves as Nonlinear Energy Sink (NES), where the acoustic power from the primary linear system is irreversibly transferred to and dissipated. The phenomenon is known as Targeted Energy Transfer (TET). Three different test rig configurations have been investigated, in order to show the effect of three different boundary conditions (BCs) of the primary system on the occurrence of TET. These BCs represent the way the primary system is connected to the excitation source and to the nonlinear attachment. Despite the TET, the acoustic waves still propagate linearly in the duct. For this reason, the classical wave decomposition technique has been used to describe the acoustic field in the primary system. The amplitude of the acoustic pressure in the duct and the reflection coefficient at the end section of it have been measured as a function of excitation amplitude and frequency in order to assess the effect of the different BCs.

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1. Introduction

Nonlinear acoustics is commonly associated with the propagation of large-amplitude disturbances, for example with plane waves with pressure amplitudes above 100 Pa (134 dB) in terrestrial reference conditions. In this sense, probably the most known equations are the Burger's equations accounting for the combined effects of non-linearity and losses in the wave propagation [1] [2].
However, nonlinear acoustics includes a number of different phenomena. For instance, another well investigated field where acoustic nonlinearities occur is that of acoustic materials provided with small apertures. In fact, several solutions have been proposed in duct and room acoustics, which exhibit nonlinear behavior when low Strouhal numbers occur in the apertures. Non-linear acoustic properties of perforates and micro-perforates have been studied in a number of research works, e.g. [3], [4], [5], [6].

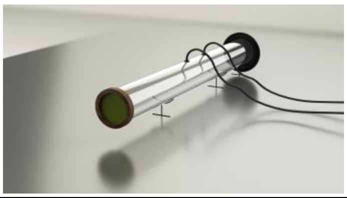
The type of nonlinearities which are object of this paper regard the fluid-structure interaction between acoustic waves propagating in linear regime and flexible structures which show non-linear dynamic behavior. In the last fifty years, researchers have studied the nonlinear acoustic behavior of viscoelastic materials, which play a major role in non-linear fluid-structure interactions. One of the pioneer study dates back to 60s', where the nonlinear response of a circular membrane to sinusoidal acoustical excitation was modeled [7].
However, the attention towards systems with essentially nonlinear attachments has been boosted by the studies, carried out in the field of non-linear mechanics, related to the so called Nonlinear Targeted Energy Transfer (TET) [8].
The TET consists of large amount of energy which is transferred from the linear system to the nonlinear attachment, also known as Nonlinear Energy Sink (NES), where the energy is finally dissipated. In mechanical systems, the linear and non-linear behaviors can be obtained by using linear and

nonlinear springs, respectively. The energy dissipation in nonlinear attachment can be ascribed to the presence of a dashpot. The occurrence of TET is strictly related to the level of excitation applied on the primary system and to the characteristics of the entire system [8] [9].
Only recently, the phenomenon of TET, previously observed and studied in mechanical systems, has been proved to occur also in acoustical systems [10]. The TET in acoustical systems has been described in terms of equivalent vibro-elastic models and different acoustic configurations have been explored with the aim of enhancing the TET phenomenon [11] [12].
The objective of the present paper is to study the occurrence of TET in acoustical configurations where the primary system (a straight duct) is connected to an excitation source (a loudspeaker) and to an essentially nonlinear attachment (a circular visco-elastic membrane) by means of different boundary conditions (BCs).
In the first part of the paper, the different BCs are shown and the methodology of investigation is explained. In the second part, the results are commented in terms of pressure drops measured in the primary system and reflection coefficient at its end section. end section.

2. Boundary conditions and methodology

In this study, the TET is studied in three different acoustical configurations. The primary linear system is represented by a straight duct where the acoustic waves, generated by an external loudspeaker, propagate in planar linear regime. The essentially non-linear attachment is represented by an elastic, circular membrane. Different boundary conditions (BCs) of the primary system are generated by attaching the loudspeaker and the membrane directly to the end section of the duct or by using one or two coupling boxes. The different BCs and the related test configurations are schematized in Table 1. Here, also rendered images of the different configurations are shown.
In order to describe the acoustic behavior of the systems, the acoustic field within the duct is studied by means of linear approach. In fact, the sound field is still linear in the primary system because the pressure disturbances are small compared to the limit of 100 Pa for the acoustic linear regime. However, in order to capture the main nonlinear effects related to the presence of the non-linear mechanical attachment, the tests have been repeated at different level of sound excitation.

Table I. Test-rig configurations implementing different BCs.

Boundary condition	Test rig	Figure
BC1	LS-Duct-Membrane	
BC2	LS-Box-Duct-Membrane	
BC3	LS-Box-Duct-Box-Membrane	

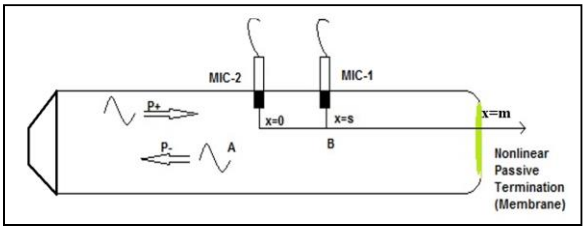


Figure 1. Schematic view of left and right moving waves in the plane wave region.

The two-microphone method has been used to describe acoustical field [13]. In the plane wave region, the Fourier Transform of the acoustic pressure in stationary medium can be described by means of the following equation:

$$p'(x) = p^{+}\, e^{-ikx} + p^{-}\, e^{+ikx} \tag{1}$$

where $k=\omega/c$ is the wave number, i.e. ratio between angular frequency and speed of sound, x is the generic position along the duct (see Fig. 1). The first term of the sum in Eq. (1) refers to the wave component moving in the positive direction (progressive wave) and the second one to the wave component moving in the opposite direction (regressive wave) (see Fig. 1). In Eq. (1), the only unknowns are p^{+} and p^{-}, thus it is sufficient to

measure the acoustic field at two specific microphone positions (A and B in Fig. 1) separated by a distance s.
The reflection coefficient at the end section of the duct ($x=m$) can be computed as for 1-port systems [14]:

$$R = \frac{p^{+}\, e^{-ikm}}{p^{-}\, e^{+ikm}} \qquad (2)$$

In order to reduce the measurements errors, it is preferable to use transfer functions in the equations above. These transfer functions are taken between the microphone signals and the reference signal represented by the electric signals driving the external source [15].
Notice that, with the approach used here, the effect of the sound radiated by the membrane itself has been neglected.

3. Experimental set-up

In all configurations examined in this work, the primary linear system remains unchanged. It is a tube with total length of 2m and inner diameter of 0.1m.
The acoustical excitation has been provided by a loudspeaker HERTZ EV165 (140W, 120mm diameter). The loudspeaker is driven by a software-based signal generator through National Instruments™ NI9269 analog output module and power amplifier (Velleman™ VPA2100MN). The signal acquisition has been performed by a dynamic signal analyzer (National Instruments™ NI NIcDAQ 9174 and NI 9234), controlled by PC based virtual instrument (LabVIEW™). The acoustic pressure has been measured by using two flush mounted ¼" pre-polarized pressure microphones (type 40BD G.R.A.S.™, equipped with preamplifiers 26CB G.R.A.S.™). The microphone separation distance is s=300mm. According to the relationship $0.1\pi<ks<0.8\pi$ [16], the corresponding frequency region reckoned during the measurements ranges from 57 Hz to 457 Hz at the temperature of 20°C. One of the two microphones (indicated as Mic.2) is located at the midsection of the duct (distance 1m from the end sections).
Details about the characteristic material of the membrane are listed in Table II and taken from manufacturer datasheet. In tests where one or two coupling cavities (expansion chambers) have been used, these one have the same cubic shape and $0.036m^3$ volume.

In Fig. 2, the amplitude of the Fourier Transform of the acoustic pressure measured by microphone located at the middle section of the duct is plotted for the three BCs. In Fig. 3, the reflection coefficient, measured at the end cross section of the duct, is shown in terms of amplitude and phase.
In both Fig.s 2 and 3, white noise excitation with low excitation level has been used, in order analyse the systems when the behaviour of the membrane is linear. At increasing of the excitation level, the trends remain unchanged until the onset of nonlinear behaviour of the membrane occurs. The peaks in pressure amplitude in Fig. 3 approximate the natural frequencies of the systems, 74 Hz for BC1, 82 Hz for BC2 and 94 Hz for BC3.
Magnitude and phase of reflection coefficients show irregular trends in the region from 50 Hz to 75 Hz due to the intrinsic characteristics of the membrane exhibited in linear regimes.
This behavior is related to the material properties and membrane pre-stress and it is independent from the characteristics of the duct. On the contrary, it will be shown that the energy dissipation related to TET occurs at the resonance frequencies of the primary system and it is dependent on the excitation level.
To investigate the occurrence of TET, a step sine excitation is used in the Sections 4 to 6, where the amplitude of the excitation is varied continuously. Each configuration is investigated in the frequency range f_n±20 Hz, where f_n is the natural frequency of the primary system. The results will be shown in terms of amplitude of the acoustic pressure measured at the midsection of the duct (Mic.2) and reflection coefficient at the end section of the duct.

Table II. Characteristics of the elastic membrane.

Membrane properties	
Diameter	100 mm
Thickness	0.5 mm
Material	Latex
Pre-stress	20 kPa
Young's Modulus	1 Mpa
Poisson's ratio	0.5

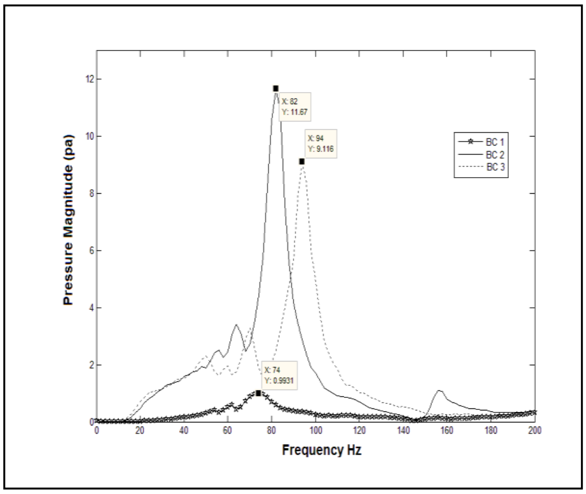


Figure 2. Pressure magnitude measured for 3 BCs by the microphone located in the middle section of the duct, under white noise excitation with low-level amplitude.

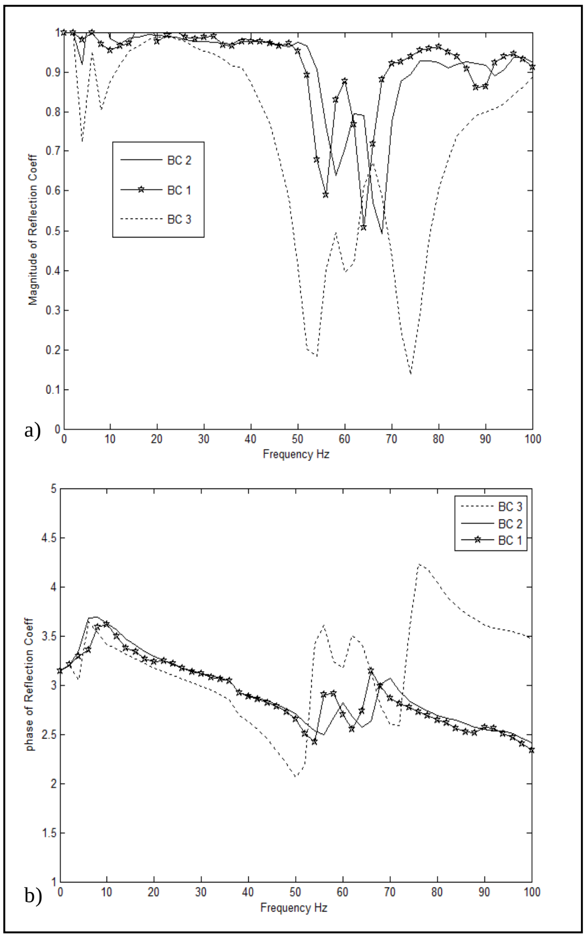


Figure 3. Reflection coefficient measured for 3 BCs at the end section of the duct under white noise excitation with low-level amplitude. a) Magnitude; b) Phase.

4.1. LS_Duct_Membrane

In the configuration with BC1, the loudspeaker is connected directly to one side of the duct and the membrane is placed on the other side (see Table I). This boundary condition is the simplest connection between nonlinear passive NES and the linear system for which f_n=74 Hz.

The results are shown in Fig.s 4 and 5. The behavior up to 74 Hz is not detailed here for brevity, since there is no occurrence of TET up to this frequency. However, it is worth mentioning that, up to 73Hz the acoustic pressure measured in the duct increases nonlinearly at the increase of the excitation. At 74 Hz the behavior appears to be linear. On the other hand, the behavior above 85 Hz is linear for the excitation range used in the tests, thus omitted in the graphs.

The Fig.s 4 and 5 show that, when the excitation reaches a certain level, the transfer of energy from the duct to the membrane increases tremendously. The acoustic energy is *pumped* from the primary system (the duct) into the nonlinear passive attachment (the membrane), which undergoes large amplitude vibrations and dissipates large amount of the incoming energy by means of viscous losses. Interestingly, for BC1 the TET occurs abruptly, resulting in steep drops in pressure and reflection coefficient taking place in short intervals of the excitation level. However, soon the system starts to slowly *recover* and both pressure and reflection coefficient begin to increase gradually and regularly.

The Fig.4 shows that the drop in acoustic pressure due to the TET takes place at frequencies from f_n+1 Hz (75 Hz) up to f_n+10 Hz (84 Hz) for the excitation ranges examined here. These drops result in local minima of the acoustic pressure ranging from 20 Pa to 27 Pa. The occurrence of TET affects also the reflection coefficient measured at the end cross section of the primary system (i.e. where the membrane is located), as shown in Fig. 5. The magnitude of the reflection coefficient drops from ≈0.95 to ≈0.45.

Remarkably, also the phase of the reflection coefficient shifts of ≈0.5π rad during the TET, as shown in Fig.5b. Outside the frequency range of TET, for further increase of the excitation level, the reflection coefficient exhibits again the values shown in Fig.3, for linear case, at different frequencies.

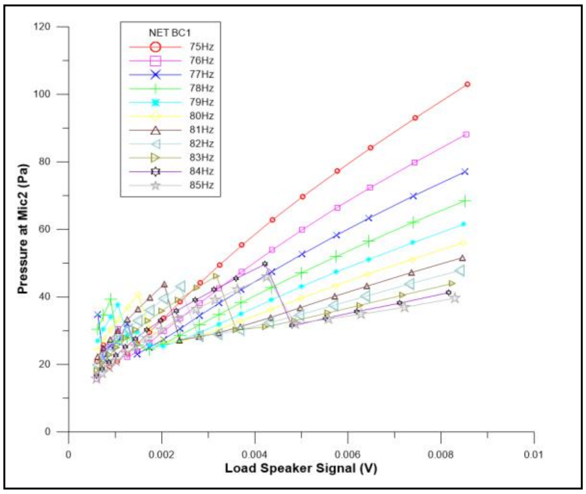


Figure 4. Pressure amplitude at the midsection of the duct (Mic.2) showing the occurrence of NET (BC1).

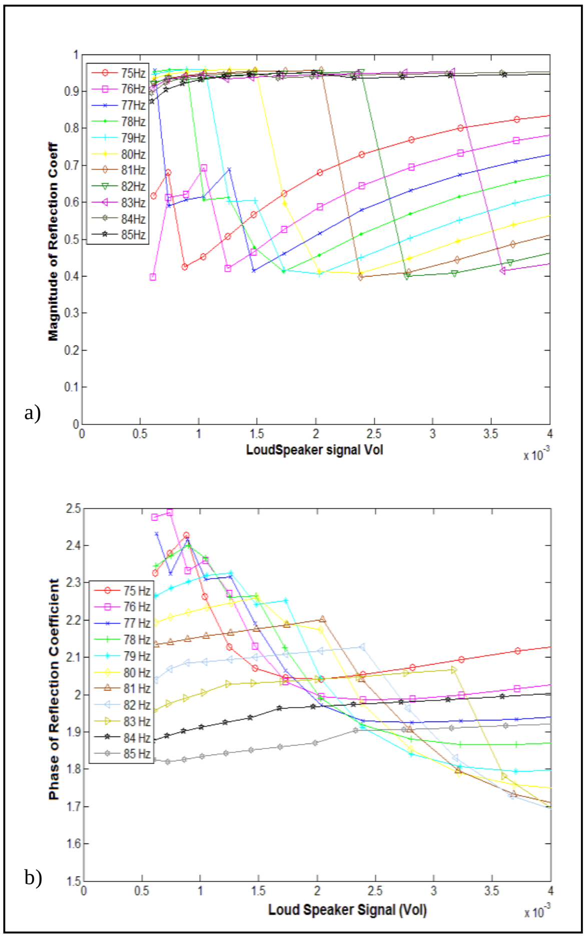


Figure 5. effect of NET in BC1 on teflection coefficient. a) Magnitude; b) Phase.

4.2. LS_Box_Duct_Membrane

In the second configuration, a coupling box is used between the loudspeaker and the duct (see Table 1). The membrane is still attached to the other termination of the duct. Now f_n=82 Hz. In terms of mechanical analogy, the coupling box acts as a soft linear spring placed between the loudspeaker and the duct.

The results are shown in Fig.s 6 and 7. Also in this case, the occurrence of TET is evident as the acoustic pressure within the duct drops despite the increase of the excitation level. On the other hand, the behavior of the system is remarkably different from what seen for BC1. For the excitation range examined, the TET occurs from f_n-3 Hz (79Hz) to f_n+7 Hz (89 Hz). Now the drop due to the TET is more gradual for both the amplitude of sound pressure and reflection coefficient. Moreover, the local minima are at ≈ 20 Pa (thus lower than in previous case), which results in even more evident reduction of the magnitude of the reflection coefficient (≈0.37). On the other hand, the behavior of the system is less predictable and all the graphs show curves with less regular trends than the ones seen in Fig.s 5 and 6. For the BC2, the phase of the reflection coefficient shifts up to ≈0.8π rad during the TET.

4.3. LS_Box_Duct_Box_Membrane

The last BC reproduces the condition tested in [10]. In this case, there are two coupling boxes, one between the loudspeaker and the duct and another one between the duct and the membrane (see Table). Here f_n=94Hz.

The results are shown in Fig.s 8 and 9. For the excitation range examined, the TET occurs from f_n-2 Hz (92Hz) to f_n+5 Hz (97Hz), thus in a quite limited frequency range. Due to the two expansion chambers confining the duct, at the resonance frequency the values of acoustic pressure reached within the duct are higher. Moreover, the minima reached at the occurrence of TET are at ≈60 Pa, instead of ≈20-27 Pa measured for BC1 and ≈20 Pa for BC2. For BC3 the decrease in magnitude of the reflection coefficient is more limited, with minima ≈0.58. However, the recover from TET is more gradual than in the two cases previously examined. The phase of the reflection coefficient shifts of ≈0.35π rad during the TET in BC3, thus less than for BC1 and BC2.

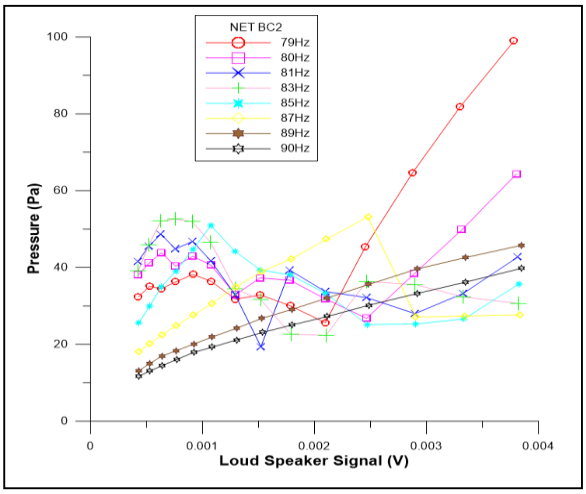


Figure 6. Pressure amplitude at the midsection of the duct (Mic.2) showing the occurrence of NET (BC2).

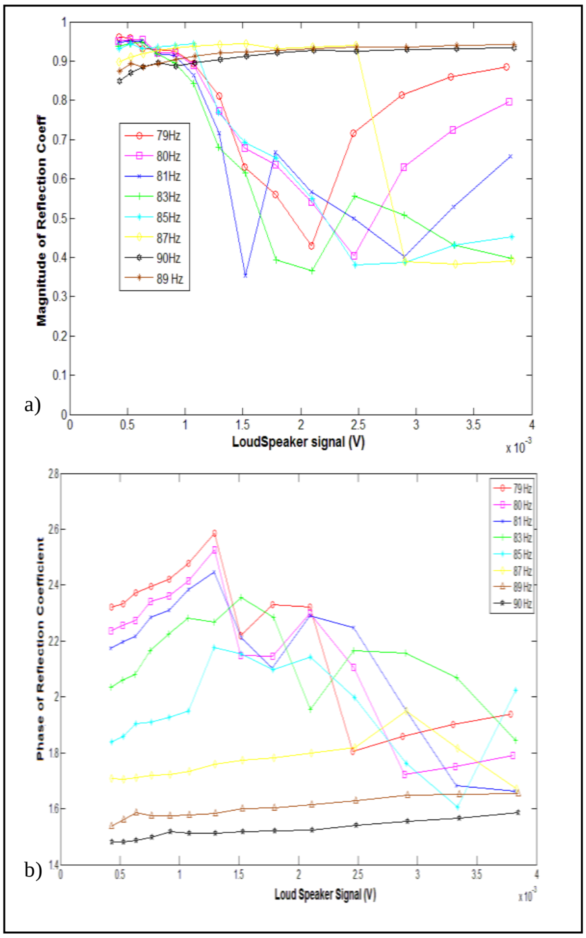


Figure 7. Effect of NET in BC2 on teflection coefficient. a) Magnitude; b) Phase.

5. Conclusions

In this paper, the phenomenon of targeted energy transfer (TET) has been experimentally studied in three different acoustic configurations. The primary linear system is represented by a straight duct where acoustic waves propagate. The essentially nonlinear

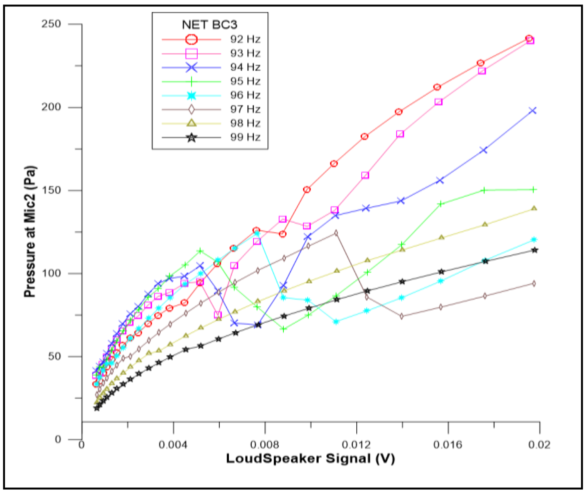


Figure 8. Pressure amplitude at the midsection of the duct (Mic.2) showing the occurrence of NET (BC3).

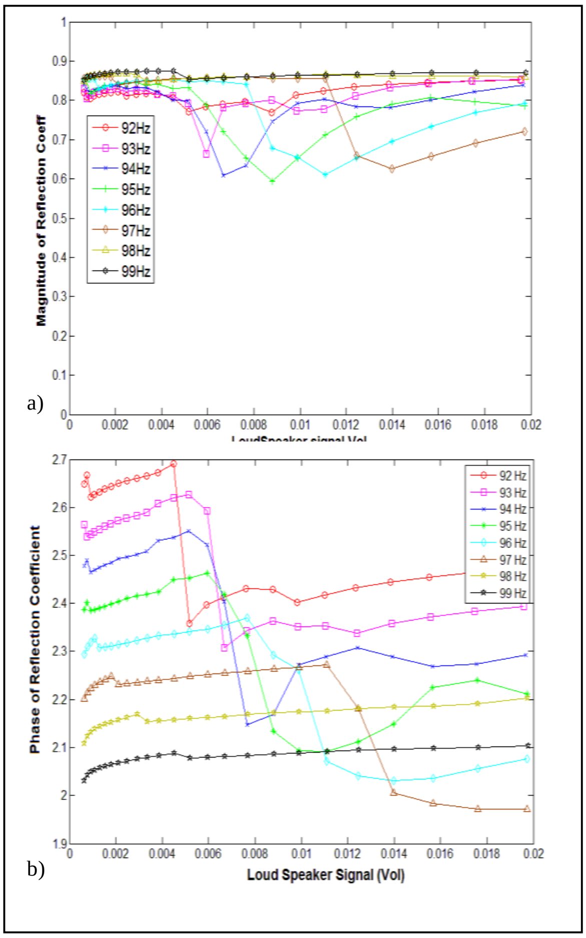


Figure 9. Effect of NET in BC3 on teflection coefficient. a) Magnitude; b) Phase.

attachment is represented by a latex circular membrane.

In order to assess the behavior of the acoustic field in the duct, the amplitude of the acoustic pressure in the midsection of the duct and the reflection coefficient at the end section of the duct have been measured under step sine excitations, with different frequencies and amplitudes.

It has been shown that the boundary conditions of the primary system heavily affect the way the TET is manifested. When the loudspeaker and the membrane are directly attached at the end sections of the duct (BC1), the TET is abrupt but the recovery is gradual and regular. When an expansion chamber is placed between the loudspeaker and the duct (BC2) the largest drops in both the acoustic pressure and the reflection coefficient are exhibited. However, the behavior of the system is less regular and predictable than in the previous case. When another expansion chamber is further added between the duct and the membrane (BC3), the acoustic pressure within the duct presents the highest values, also in presence of TET, the magnitude of the reflection coefficient shows the smallest reductions and the TET is exhibited in small frequency ranges. However, the system undergoes gradual recover from TET at the increase of the excitation level. During the TET, the phase of the reflection coefficient varies of ≈0.5 π rad for BC1, up to 0.8π rad for BC2 and of ≈0.3 π rad for BC3.

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