Semi-analytical modelling for the design of railway floating slab

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Summary
This paper aims at presenting a 3D semi-analytical model of a railway platform coupled to the ground through an elastomeric matetrial. The model is based on the bending plate vibration of the railway platform coupled to a surface spring which represents the elastomeric mat. The upper structure of the platform is excited by a punctual force while the lower structure is coupled to the ground. The problem uses a 2D spatial Fourier Transform. When sizing the characteristics of the elastomeric mat, one often considers the resonance frequency as the most important parameters. From a practical point of view for engineer, this parameter gives an average of performance insulation at higher frequency. However the resonance frequency may vary from on site to another mainly due to ground characteristics. This paper introduces the effect of such parameter on the insertion loss.

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1. Introduction

In recent decades, number of railway infrastructures grew up such as high-speed railways, trams and subways. All of these infrastructures are more and more often placed in urban area with strong acoustic and vibratory requirements. These infrastructures are generally coupled to the ground and therefor at the origin of the propagation of waves in the ground. These waves generate the vibration of buildings near railway infrastructure and are a source of significant noise pollution for residents. In this context, one of the strongest issues is to reduce the vibrations transmitted to the ground.
There are currently different systems to reduce the vibration from the train. Three major categories could be distinguished to mitigate railway vibrations. The first category is the mitigation at the source, i.e. in the vicinity of the wheel-rail interaction [1]. Without being exhaustive about all existing techniques, maintenance operations performed on the wheels of trains and rails may contribute to the reduction of vibrations. In addition, pads between the rail and the concrete slab can perform mitigation. The second category of mitigation is at the propagation path. Thus trenches in the ground depth can be made near the track and thus reduce the propagation [2]. Horizontal Vibration Barrier system has recently been developed using techniques similar to those used in this paper [3]. A slab is placed at the ground surface and blocks the vibrations. The third category of mitigation is at the reception in the building where it is possible for example to isolate buildings with springs or pads.
The purpose of this article is to provide an analytical formulation of the ground/slab coupling in the case of a floating slab. A situation often encountered in urban area is modelled with the aim of mitigating the propagation of the vibration and understand the ground effect. In particular, it will be shown that the soil has a significant effect on the performance of a resilient.

2. Floating slab modelling subjected to a punctual load force

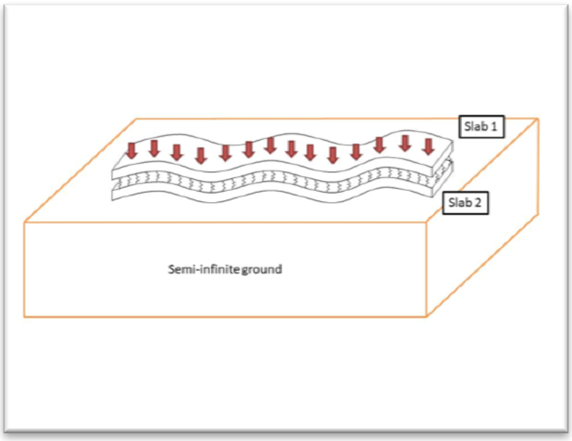

Figure 1 - Overview of the problem.

In this section, the problem of two slabs coupled together by a spring is modelled. One slab called slab 2 is coupled to the ground. Figure 1 gives an overview of the problem under consideration.[4]

2.1. Slabs coupled to the ground

Two finite slabs called 1 and 2 are modelled using the Kirchhoff-Love hypothesis so we neglect the shear deformation and the rotary inertia. In the frequency domain, the equations of motion of the two slabs are:

$$\begin{cases} D_1^*\nabla^4 w_1 - \rho_1 h_1 \omega^2 w_1 = F_0 \delta(x - x_0)\delta(y - y_0) \\ \qquad + K_t^*(w_2 - w_1) \\ D_2^*\nabla^4 w_2 - \rho_2 h_2 \omega^2 w_2 = \sigma_p + K_t^*(w_1 - w_2) \end{cases}$$

(1)

Where $D_1^* = D_1(1 + j\eta_1)$ and $D_2^* = D_2(1 + j\eta_2)$ are complex flexural stiffness. F_0 is the amplitude of the force applied on the plate 1 to the point $(x_0; y_0)$. The coupling between the plate 1 and the plate 2 is represented by a spring of stiffness $K_t = \frac{E_t^* S_t}{h_t}$ and damping η_t. σ_p represents the stress due to the ground/slab coupling.

The unknowns of the problem are the displacements of plates 1 and 2 which can be expanded in series of slabs modes:

$$\begin{cases} w_1 = \sum_{nm} a_{nm} \phi_{nm} \\ w_2 = \sum_{pq} b_{pq} \phi_{pq} \end{cases} \qquad (2)$$

where a_{nm} and b_{pq} are the modal amplitude and ϕ_{nm} and ϕ_{pq} are the modal shape of the plate 1 and 2 respectively.

In this problem we consider the guided boundary conditions where the shear force and the rotation are null at the slab boundaries. This allows providing a simple series over cosines functions while taking into account the rigid body of first order in this kind of study. So we have:

$$\begin{cases} \phi_{nm} = cos\left(\frac{n\pi}{L_{x1}}\,x\right) cos\left(\frac{m\pi}{L_{y1}}\,y\right) \\ \phi_{pq} = cos\left(\frac{p\pi}{L_{x2}}\,x\right) cos\left(\frac{q\pi}{L_{y2}}\,y\right) \end{cases} \qquad (3)$$

A modal series of the force is also carried out for regularization of the problem:

$$F(x, y) = \sum_{nm} F_{nm} \phi_{nm} \qquad (4)$$

We replace the expression of the slab displacement (2) and the force (4) in the equations of the movement (6) and which gives:

$$\begin{cases} \sum_{nm}\big((D_1^* k_{nm}^4 - \rho_1 h_1 \omega^2) a_{nm} - F_{nm}\big)\phi_{nm} \\ \quad = K_t^*\big(\sum_{pq} b_{pq}\phi_{pq} - \sum_{nm} a_{nm}\phi_{nm}\big) \\ \sum_{nm}\big(D_2^* k_{pq}^4 - \rho_2 h_2 \omega^2\big) b_{pq}\phi_{pq} = \sigma_p \\ \quad + K_t^*\big(\sum_{nm} a_{nm}\phi_{nm} - \sum_{pq} b_{pq}\phi_{pq}\big) \end{cases}$$

(6)

2.2. Ground modelling

The ground is modelled with Navier's equation which considers a continuous, homogeneous and isotropic elastic medium:

$$\mu \nabla^2 \vec{u} + (\mu + \lambda)\nabla(\nabla.\vec{u}) + \rho\omega^2 \vec{u} = \vec{0} \quad (7)$$

where $\vec{u}^T = \{u_x, u_y, u_z\}$ is the vector of ground displacement.

The ground is a semi-infinite medium in z-direction and infinite in the direction x- and y-direction. Tangential stresses along x- and y-direction with respect to the z are zero at the surface. Normal stress along the z axis is also zero everywhere on the surface z = 0 except under slabs 2:

$$\begin{cases} \sigma_{xz}(x, y, 0) = 0 \;\forall (x, y) \in \mathbb{R}^2 \\ \sigma_{yz}(x, y, 0) = 0 \;\forall (x, y) \in \mathbb{R}^2 \\ \sigma_{zz}(x, y, 0) = \begin{cases} \sigma_2(x, y) \;\forall (x, y) \in S_2 \\ 0 \;\forall (x, y) \in \mathbb{R}^2 - S_2 \end{cases} \end{cases} \qquad (8)$$

where σ_{xz} and σ_{yz} represent tangential stresses and σ_{zz} represent normal stress. σ_2 represents the stress applied to the slab 2 by the ground.

It is also necessary to consider the continuity of displacements at the interface between the slab 2 and the ground:

$$w_2(x, y) = u_z(x, y, 0) \forall (x, y) \in S_2 \qquad (9)$$

2.3. Ground / structure coupling resolution using 2D Fourier Transform and modal decomposition

Since the ground is infinite in the x and y directions, a solution of the ground displacement is given by 2D Fourier spatial transform.
A well-known technique to solve the equations of motion (7) consists in using a Helmholtz decomposition showing two categories of volume waves, the shear wave c_s and the dilatational waves c_p. The boundary conditions at the ground surface in the Fourier domain are performed. The tangential stresses σ_{xz} and σ_{yz} is null in the Fourier domain whereas the normal stress σ_{zz} requires the coupling with the slab 2 to be taken into account:

$$\tilde{\sigma}_{zz}\left(k_x,k_y,0\right) = \iint_{S_2} \sigma_{zz}(x,y,0)e^{-j\left(k_x x+k_y y\right)}dS = \tilde{\sigma}_2\left(k_x,k_y\right) \qquad (13)$$

where $\tilde{\sigma}_2$ represents the contribution of the 2D Fourier transform of the normal stress applied to the slab 2 by the ground.
The expression of the ground displacement at the top surface in the Fourier domain can be put in the following form:

$$\tilde{u}_{zz}\left(k_x,k_y,0\right) = N\left(K_x,k_y\right)\tilde{\sigma}_2\left(k_x,k_y\right) \quad (14)$$

2.4. Solution for the plate modal amplitude

The unknowns of this problem are the modal amplitudes of the slab 1 and 2, that is to say: a_{nm} and b_{pq}. The condition of continuity of displacements between the plate 2 and the ground is used:

$$u_z(x,y,0) = w_2(x,y)\ \forall(x,y) \in S_2 \qquad (15)$$

The displacement of the ground is obtained by taking the inverse Fourier transform of (14). In addition we use the relation (6) that we put in the expression (13) to finally obtain:

$$\frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} N\left(k_x,k_y\right)\Big(\sum_{pq}\left(D_2^* k_{pq}^4 - \omega^2\rho_2 h_2\right)b_{pq}\phi_{pq} - K_t^*\sum_{nm} a_{nm}\phi_{nm} + K_t^*\sum_{pq} b_{pq}\phi_{pq}\Big)e^{j\left(k_x x+k_y y\right)}dk_x dk_y = \sum_{pq} b_{pq}\phi_{pq} \qquad (16)$$

Once a multiplication by the modal shape 2 and an integration over the slab 2 surface, we obtain:

$$\sum_{pq}\left(D_2^* k_{pq}^4 - \omega^2\rho_2 h_2 + K_t^*\right)b_{pq}\gamma_{pqtu}^{22} - K_t^*\sum_{nm} a_{nm}\gamma_{nmtu}^{21} = b_{tu}S_{tu} \qquad (17)$$

with $\gamma_{pqtu}^{22} = \iint_{-\infty}^{+\infty} N\left(k_x,k_y\right)\tilde{\phi}_{pq}\phi_{tu}^* dk_x dk_y$ and $\gamma_{nmtu}^{21} = \iint_{-\infty}^{+\infty} N\left(k_x,k_y\right)\tilde{\phi}_{nm}\phi_{tu}^* dk_x dk_y$.
Similar as for equation (17) we multiply the slab 1 equation of motion by the modal shape 1 and an integration over the slab 1 surface is performed to obtain:

$$\sum_{nm}\left(\left(D_1^* k_{nm}^4 - \omega^2\rho_1 h_1 + K_t^*\right)a_{nm} - F_{nm}\right)\gamma_{nmrs}^{11} - K_t^*\sum_{pq} b_{pq}\gamma_{pqrs}^{12} = 0 \qquad (19)$$

With $\gamma_{nmrs}^{11} = \iint_{S_1} \emptyset_{nm}\emptyset_{rs}dS$ and $\gamma_{pqrs}^{12} = \iint_{S_1} \emptyset_{pq}\emptyset_{rs}dS$.
The linear system to evaluate is:

$$\begin{cases} \sum_{nm}\left(\left(D_1^* k_{nm}^4 - \omega^2\rho_1 h_1 + K_t^*\right)a_{nm} - F_{nm}\right)\gamma_{nmrs}^{11} \\ \qquad -K_t^*\sum_{pq} b_{pq}\gamma_{pqrs}^{12} = 0 \\ \sum_{pq}\left(D_2^* k_{pq}^4 - \omega^2\rho_2 h_2 + K_t^*\right)b_{pq}\gamma_{pqtu}^{22} \\ \qquad -K_t^*\sum_{nm} a_{nm}\gamma_{nmtu}^{21} - b_{tu}S_{tu} = 0 \end{cases} \qquad (21)$$

which gives in matrix format:

$$\begin{pmatrix} \gamma_{nmrs}^{11} & (0) \\ (0) & \gamma_{pqtu}^{22} \end{pmatrix} \begin{pmatrix} D_1^* k_{nm}^4 - \omega^2\rho_1 h_1 & (0) \\ (0) & D_2^* k_{pq}^4 - \omega^2\rho_2 h_2 \end{pmatrix}\begin{Bmatrix} a_{nm} \\ b_{pq} \end{Bmatrix} + K_t^*\begin{pmatrix} \gamma_{nmrs}^{11} & -\gamma_{pqrs}^{12} \\ -\gamma_{nmtu}^{21} & \gamma_{pqtu}^{22} \end{pmatrix}\begin{Bmatrix} a_{nm} \\ b_{pq} \end{Bmatrix} - \begin{pmatrix} (0) & (0) \\ (0) & S_{tu} \end{pmatrix}\begin{Bmatrix} a_{nm} \\ b_{pq} \end{Bmatrix} = \begin{pmatrix} \gamma_{nmrs}^{11} & (0) \\ (0) & \gamma_{pqtu}^{22} \end{pmatrix}\begin{Bmatrix} F_{nm} \\ (0) \end{Bmatrix} \qquad (22)$$

In the present case, the number of mode of the slab 1 and 2 will be identical as well as the surface so $\gamma_{nmtu}^{21} = \gamma_{pqtu}^{22}$.
It is now possible to give the expression of the modal amplitude by solving the linear system above. We are also interested to determine the displacement at the ground surface. The modal amplitude of the slabs allows to determine the ground displacement by carrying out an inverse 2D Fourier transform of expression (14):

$$u_z(x,y,0) = \sum_{pq}\left(D_2^* k_{pq}^4 - \omega^2\rho_2 h_2 + K_t^*\right)b_{pq}T_{pq}(x,y) - K_t^*\sum_{nm} a_{nm}T_{nm}(x,y) \qquad (24)$$

with
$T_{pq}(x,y) = \frac{1}{4\pi^4}\iint_{-\infty}^{+\infty} N\left(k_x,k_y\right)\tilde{\phi}_{pq}e^{j\left(k_x x+k_y y\right)}dk_x dk_y$ and

$T_{nm}(x,y) = \frac{1}{4\pi^4}\iint_{-\infty}^{+\infty} N(k_x,k_y)\,\tilde{\Phi}_{nm} e^{j(k_x x+k_y y)} dk_x dk_y$

3. NUMERICAL RESULTS

The modelling that is carried out here consists of an isolation mat commonly used for tramway platforms in order to mitigate vibration.

For the sake of simplification in this study, only homogeneous semi-infinite ground is considered although the case of a stratified ground may be modelled. The ground that is considered has the following characteristics: $c_p = 700\,m.s^{-1}$, $c_s = 300\,m.s^{-1}$, $\eta_p = 2\%$, $\eta_s = 2\%$ and $\rho = 2000Kg.m^{-3}$. The slabs are a concrete slabs ($E_p = 2.5 * 10^{10}Pa$, $\rho_p = 2500Kg.m^{-3}$, $\eta_p = 5\%$).

The performance of the mat is given using the insertion loss, i.e the levels ratio with and without resilient mat. We can therefore formulate the insertion loss in dB by the following formula:

$$IL_{moy} = 10.\log_{10}\left(\frac{|u_{z\,without\,mat}|^2}{|u_{z\,with\,mat}|^2}\right) \qquad (25)$$

3.1. Rigid mass case

One considers a small rectangular slab of 6m where the bending effects are initially neglected. Only the rigid body is taken into account which reduces the linear system (22) to the following system:

$$\begin{pmatrix} \omega^2\rho_1 h_1 & 0 \\ 0 & \omega^2\rho_2 h_2 \end{pmatrix}\begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} + \begin{pmatrix} 1 & -1 \\ -K_t^* & K_t^* - \frac{S}{\gamma_{0000}} \end{pmatrix}\begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \qquad (26)$$

We recognize here a system with two degrees of freedom where the coupling with the ground is represented by the term $-\frac{S}{\gamma_{0000}}$. A mass-spring-mass system has a frequency of resonance of $f_{res} = \frac{1}{2\pi}\sqrt{K_t\left(\frac{1}{\rho_1 h_1}+\frac{1}{\rho_2 h_2}\right)}$. The resilient that we consider has a Young's modulus of 10 000 Pa and thickness 3cm. Its stiffness is then $K_t = 12MN.m^{-1}$ for a surface area of 36m². The aim here is to know if the coupling with the ground has an effect on the resonance frequency. The thickness of the upper slab 1 and lower slab 2 is 20cm. Figure 2 shows the transmission loss in the case where the slab 2 is not coupled to the ground and coupled to the ground. The transmission loss in the case of coupling with the ground is very important due to the strong ground / structure coupling. In this case there is no more amplification.

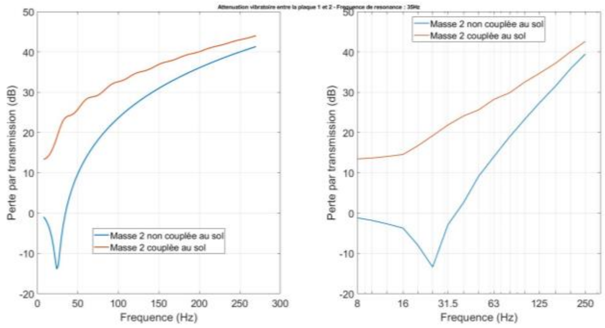

Figure 2 : Transmission loss between the slab 1 and 2 with and without ground coupling

We are interested in the insertion loss on the lower slab 2 that is to say the slab coupled to the ground. The insertion loss corresponds to the difference of level on the slab 2 with and without resilient mat. Figure 3 shows the insertion loss in the case of a slab which is not coupled to the ground and coupled to the ground. When the slab 2 is not coupled to the ground, the amplification of the system is at 27 Hz. On the other hand, when the slab 2 is coupled with the ground, this resonance frequency shifts in high frequency. This phenomenon is due to the ground added stiffness to the slab represented by the term $-\frac{S}{\gamma_{0000}}$ in equation (26). It should be noted that this term is variable with the frequency consequently the added rigidity to the system vary. In addition, the added stiffness by this term tends to 0 while increasing in frequency which is translated on the curves of Figure 3.

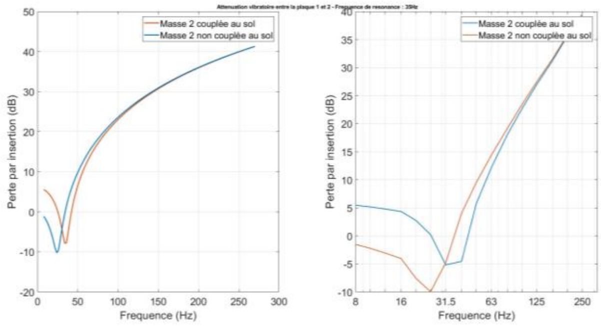

Figure 3 : Insertion loss between the slab 1 and 2 with and without ground coupling

3.2. Performance of the resilient depending on the thickness of slabs

We are interested in the variation of the thickness of slabs 1 and 2 on the insertion loss. We no longer consider the slab 2 of identical thickness for the determination of the insertion loss. In the case of a slab without resilient mat, the thickness is 60cm. From a practical point of view when we estimate an insertion loss of a tramway railway platform, the floating slab consists of two slabs of variable thickness while in the case of a classic slab we consider usually a slab thickness slab of 60cm

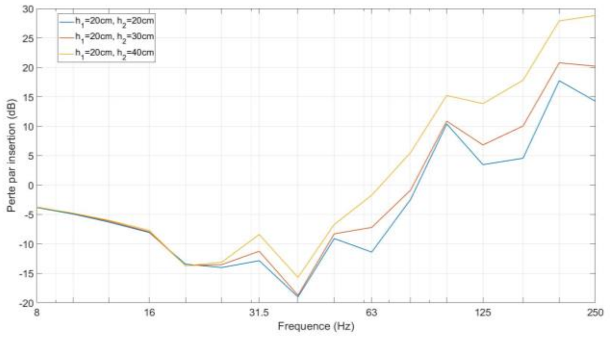

Figure 4 - Insertion loss for different thickness of slab 2 (20cm, 30cm and 40cm)

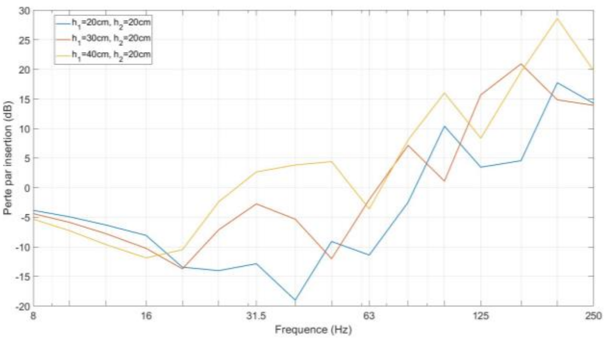

Figure 5 - Insertion loss for different thickness of slab 1 (20cm, 30cm and 40cm)

3.3. Performance of the resilient depending on the stiffness of the mat

We now consider the case of a tramway slab of 20m long and 3m length. The force applied to the slab is a sum of uncorrelated force [5]. We look at the insertion loss between the level on the slab 2 coupled to the ground of 20cm and a classical slab coupled to the ground of 60cm. The Young's modulus of the resilient remains unchanged and is 10 000Pa.

Figure 6 corresponds to the insertion loss for different thickness of the elastomeric layer. The insertion loss is calculated at the ground top surface at 10m from the tramway slab. As the thickness of the layer is higher, the performance of isolation is better. The resonance frequency for the layer 3cm thick reaches 14dB at 80Hz however these performances are not perfectly similar if we look at the slab vibration. This is due to local behaviour of the ground.

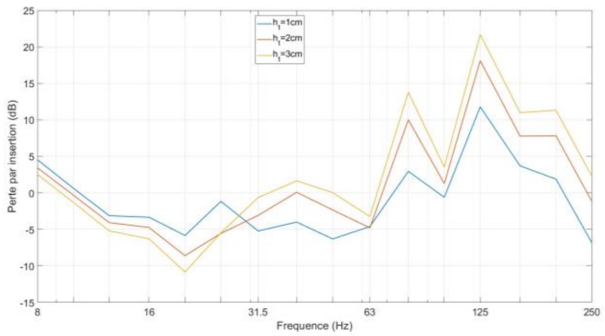

Figure 6 Insertion loss at the ground top surface at 10m from the tramway slab for different thickness of the elastomeric layer ($h_t = 1cm,\ 2cm,\ 3cm$)

3.4. Influence of mechanical properties of the ground on resilient performances

In this last section, we are interested with the influence of the ground on the performance of an elastomeric layer. We are still considering a tramway slab excited by a sum of uncorrelated force. We look at the insertion loss between the level for an isolated tramway slab (lower slab 20cm, upper slab 60cm) and a classical slab coupled to the ground (60cm).

Figure 7 shows the insertion loss on the slab for four different ground characteristics which are $c_s = 200m.s^{-1}$, $c_s = 300m.s^{-1}$, $c_s = 400m.s^{-1}$ and $c_s = 500m.s^{-1}$. As expected the resonance frequency at 25Hz for the softer ground shift to 31.5Hz for stiffer ground. Close to the resonance frequency the performance may have large difference value however at higher frequency the performance is not influence by the ground characteristics.

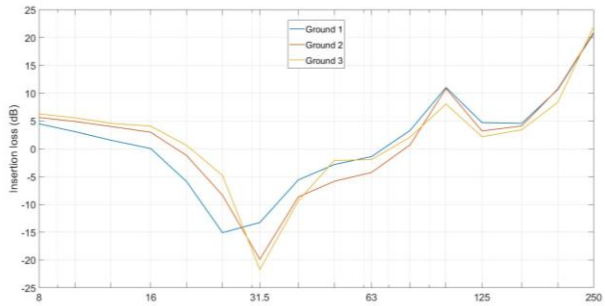

Figure 7 : Insertion loss on the tramway slab for different ground characteristics

4. CONCLUSION

We have presented in this paper an analytical modelling of a lofting slab which account for the flexural vibration for the structure as well as its rigid body. A parametric study shows that the ground has an influence on the resonance frequency of the system and should be considered while sizing a tramway floating slab. It has also been shown that the size of the floating slab has an important effect on the performance compared to the size of the lower slab.

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