Adjustability of acoustic properties of surfaces at low frequencies by an array of active resonators

Nikolay Kanev
Andreyev Acoustics Institute, Russia.

Summary

A concept of surfaces with actively controlled acoustic properties is presented in this paper. An array of secondary sound sources is placed near the flat surface with arbitrary acoustic properties or mounted onto the surface. Each secondary source is an active resonator, which is an electromechanical device consisting of a loudspeaker connected with a microphone by a feedback. The acoustic impedance of the loudspeaker is actively controlled in wide frequency band. It is shown that the array may absorb, reflect or scatter incident sound waves in dependence on adjustment of the active resonators. Proposed approach is very effective at low frequencies when a distance between the active resonators is smaller a half wavelength. In some cases efficiency of active control can be good enough if the distance is not greater the wavelength. Absorption and reflection coefficients for a plane wave incident on the surface with actively controlled properties are found.

PACS no. 43.50.Ki, 43.20.Fn

1. Introduction

Surfaces with variable acoustic properties are useful for many applications. For example, in multipurpose concert halls adjustable acoustic treatment is often installed to accommodate different types of performances. There are many solutions changing acoustics of the room by means of removable panels or draperies [1]. These solutions provide sufficient flexibility in room acoustics at medium and high frequencies, whereas variations at low frequencies are not significant. There are special absorbers with variable absorption coefficient for low frequency control [2].

New opportunities have appeared with development of active sound control methods [3]. In the last few decades many approaches and strategies have been proposed and discussed. In contrast to passive systems, the active noise control systems are much smaller and lighter for low frequency applications. Some of them deal with acoustical impedance control [4,5].

The simplest device for active sound absorption consists of a loudspeaker and a sensor connected by a feedback [6]. If the active control of loudspeaker motion provides the resonant impedance of the loudspeaker in a wide frequency band, the device is named an active resonator. The active resonators were designed and experimentally investigated in different application some years ago [7-9]. Note that control the acoustic impedance of the loudspeaker can be realized without the sound field sensor [10].

In this paper the array of the active resonators is considered. Passive resonators array may effectively absorb and scatter sound waves [11-12] in narrow frequency band. The active resonator array works in similar way but in wide frequency band.

First of all we briefly outline the concept of the active resonator for sound control. Then we apply it for an infinite flat array of the active resonators placed near a surface with arbitrary acoustic properties and find conditions for maximal absorption, reflection and scattering of the sound waves.

2. Active resonator

In practice resonators are often used for noise control in many applications. They can effectively absorb or scatter sound waves. The general condition of maximal absorption by the resonator is given by

$$\operatorname{Re} Z = \operatorname{Re} Z_r ,\ \operatorname{Im}(Z + Z_r) = 0 , \qquad (1)$$

where Z is the mechanical impedance of resonator, Z_r is the radiation impedance.

The general condition for maximal scattering is

$$\operatorname{Re} Z = 0,\ \operatorname{Im}(Z + Z_r) = 0. \qquad (2)$$

The conditions (1) and (2) can be accomplished only at resonant frequency. So the main disadvantage of the resonators is narrow frequency band, where they effectively control the sound waves.

In order to use the significant feature of resonator the active control strategy based on realization of resonant condition for a secondary source was proposed [9]. If the impedance of the secondary source satisfies the condition (1) or (2) in wide frequency band the secondary source will absorb or scatter the maximal value of sound power.

The scheme of the active resonator is shown in Fig. 1. It consists of the loudspeaker and the microphone placed near the radiating surface of the loudspeaker. The loudspeaker is connected with the microphone by means of a feedback characterizing by a feedback coefficient K depending on the frequency ω. Adjusting the coefficient K proposes to regulate the impedance of the loudspeaker.

The problem is to find the impedance of the loudspeaker under the feedback control. The loudspeaker is considered as a monopole source. Without the feedback the equation of loudspeaker motion can be written in the simplest form

$$ZQ = P_0 - Z_r Q, \qquad (3)$$

where Q in the volume velocity of the loudspeaker, P_0 in the pressure of the primary field, Z is the electromechanical impedance of the loudspeaker. The pressure measured by the microphone is $P_1 = P_0 - Z_r Q$. The signal from the microphone forms additional force acting on the loudspeaker. In general case this force is equal $P_1 K$ and has to be added in right part of the equation (3)

$$ZQ = P_0 - Z_r Q + P_1 K. \qquad (4)$$

Now motion of the loudspeaker is controlled by the feedback. From (4) its volume velocity can be found

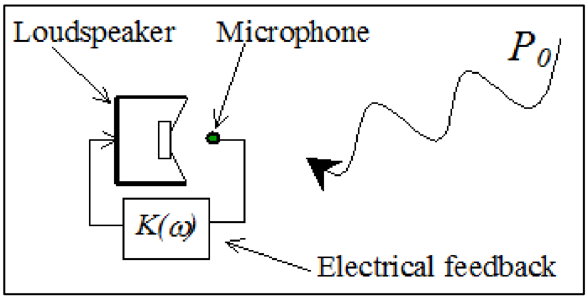

Figure 1. The scheme of the active resonator.

$$Q = \frac{P_0(1+K)}{Z + Z_r(1+K)}. \qquad (5)$$

The impedance of the loudspeaker with the feedback is

$$Z_a = \frac{P_1}{Q} = \frac{Z}{1+K}. \qquad (6)$$

If the feedback coefficient K is adjusted to satisfy the resonant conditions of maximal absorption (1) and scattering (2) then the device shown in Fig.1 behaves like the resonator, therefore it is called the active resonator. It is important that the resonant conditions can be realized in the wide frequency band.

In this paper realization the feedback and design of the active resonator are not investigated. We suppose that the active resonators are characterized by the impedance Z_a in the required frequency band.

3. Active resonator array

Under certain conditions an infinite system of resonators may absorb or scatter sound waves [11,12]. If the system is placed near any surface, the acoustic properties of the surface are significantly changed [13]. This effect is exemplified here by a regular flat array of the active resonators placed near the flat surface.

A surface with an impedance Z coincides with the plane $z = 0$ as shown in Fig. 2. A plane harmonic sound wave incident on the plane from the halfspace $z > 0$ is reflected by the surface. The incident and reflected waves are shown by red and blue arrows in Fig. 2. So the primary sound field is a sum of incident and reflected sound waves and described by the pressure

$$P_I = e^{ik_x x + ik_y y - ik_z z} + V(k_z) e^{ik_x x + ik_y y + ik_z z}, \qquad (7)$$

$$V(k_z) = \left(\frac{Z}{\rho\omega} k_z - 1\right) \Big/ \left(\frac{Z}{\rho\omega} k_z + 1\right), \qquad (8)$$

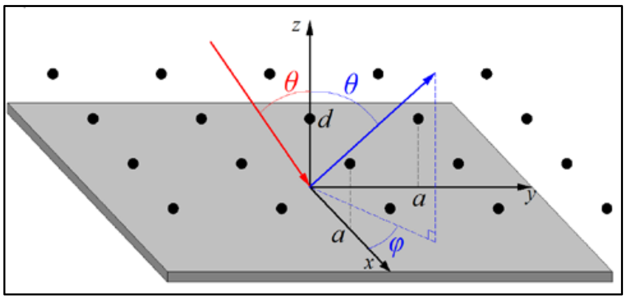

Figure 2. The active resonator array near an impedance surface.

where k_x, k_y, k_z are the projections of the wave vector of the incident wave on the *x*, *y* and *z* axes respectively, V is reflection coefficient, ω is the frequency of sound, ρ is the density of the medium. To characterize the direction of the waves it is possible to use two angles θ and φ, as shown in Fig. 2. These angles are connected with the projections of the wave vector by relations $k_x = \sin\theta\cos\varphi$, $k_y = \sin\theta\sin\varphi$, $k_z = k\cos\theta$.

Active resonators are placed at the points with coordinates $x_s = sa$, $y_q = qa$, $z = h$, where a is the array period along the *x* and *y* axes, *q* and *s* are arbitrary integers. All active resonators are identical and characterized by the impedance $Z_a = P/Q$, which is the ratio of the sound pressure *P* near the resonator to its volume velocity *Q*.

In [13] the secondary field produced by the active resonators is found. The field of the active resonator array under the primary field (7) can be written in the following form

$$P_{II} = -\frac{\omega\rho}{2a^2(Z_a + Z_r)}\times \times\left(e^{-ik_z h} + V(k_z)e^{ik_z h}\right)\sum_{n,m} P_{n,m}, \quad (9)$$

$$P_{n,m} = \frac{1}{k_z^{nm}}\left(e^{ik_z^{nm}(z-h)} + V(k_z^{nm})e^{ik_z^{nm}(z+h)}\right)\times \times e^{ik_x^n x + ik_y^n y}, \quad (10)$$

where $k_x^n = k_x + 2\pi\frac{n}{a}$, $k_y^m = k_y + 2\pi\frac{m}{a}$, $k_z^{nm} = \sqrt{k^2 - \left(k_x^n\right)^2 - \left(k_y^m\right)^2}$, $Z_r \approx \frac{\omega\rho}{2a^2}\sum_{n,m}\frac{1}{k_z^{nm}}\left(1 + V(k_z^{nm})e^{2ik_z^{nm}h}\right)$ is the radiation impedance of the active resonator.

The summation in (10) is performed over all integer *n* and *m*.

We can see that the secondary field consists of plane waves, which can be characterized by two integers (*n*,*m*) and wave vector $\left(k_x^n, k_y^m, k_z^{nm}\right)$. Each wave (10) is a superposition of the radiated wave and reflected one from the surface $z = 0$. The wave (*n*,*m*) is uniform and propagates in the halfspace $z > h$ if $k^2 \geq \left(k_x^n\right)^2 - \left(k_y^m\right)^2$. In opposite case $k^2 < \left(k_x^n\right)^2 - \left(k_y^m\right)^2$ the wave (*n*,*m*) is nonuniform, it is not radiated in the halfspace $z > h$.

Note that the wave vector $\left(k_x^0, k_y^0, k_z^{00}\right)$ of the wave (0,0) coincides with the wave vector $\left(k_x, k_y, k_z\right)$ of the reflected primary field (7). It means that under certain conditions the wave (0,0) can cancel the reflected wave due to interference.

Absorption of the incident wave takes place if the amplitudes of the reflected wave (7) and the secondary wave are equal and their phases are opposite. It is useful when the surface has low sound absorption properties because it is possible to increase absorption by means of the active resonators. So active control of the absorption coefficient can be realized by the considered system.

For the absorbing surface it is possible to change the reflected field as well. The amplitude of the primary reflected wave (7) is small. If the amplitudes of the incident wave and the secondary wave (0,0), are equal the absorption coefficient is zero. The incident wave is totally reflected by the surface with the active resonators array.

With increase of frequency the waves with non-zero (*n*,*m*) can be radiated by the active resonator array. It depends on the array period a and the incident angle θ.

Fig. 3 shows the wave vectors of the radiated waves for different incident waves and the array period described by the dimensionless parameter *ka*. Three directions of the incident waves are shown: $\theta = 0$ (normal incidence); $\theta = \pi/4$ and $\varphi = \pi/2$; $\theta = \pi/4$ and $\varphi = \pi/4$. Three periods $ka = 0.5$; 1.5; 2.5 are considered as well. The red arrows are incident waves, the blue arrows are the reflected waves and the wave (0,0) radiated by the active resonators, the green lines are radiated non-zero waves (*n*,*m*) or Bragg waves.

If the array period is smaller a half wavelength, the secondary field consists of only one wave (0,0). If the period is not grater a wavelength, radiation of non-zero waves depends on the incident angle. At normal incidence and the period $ka = 1.5$ only wave (0,0) is radiated, whereas at oblique incidence Bragg waves are radiated as well. Such waves are always radiated if the period is greater a wavelength.

Existing of Bragg waves is crucial for active sound control because they radiate sound energy in different directions and they do not coincide with the primary waves. It means that the secondary field is not suitable for useful interference with the primary field. Therefore it is a fundamental limitation of the proposed method.

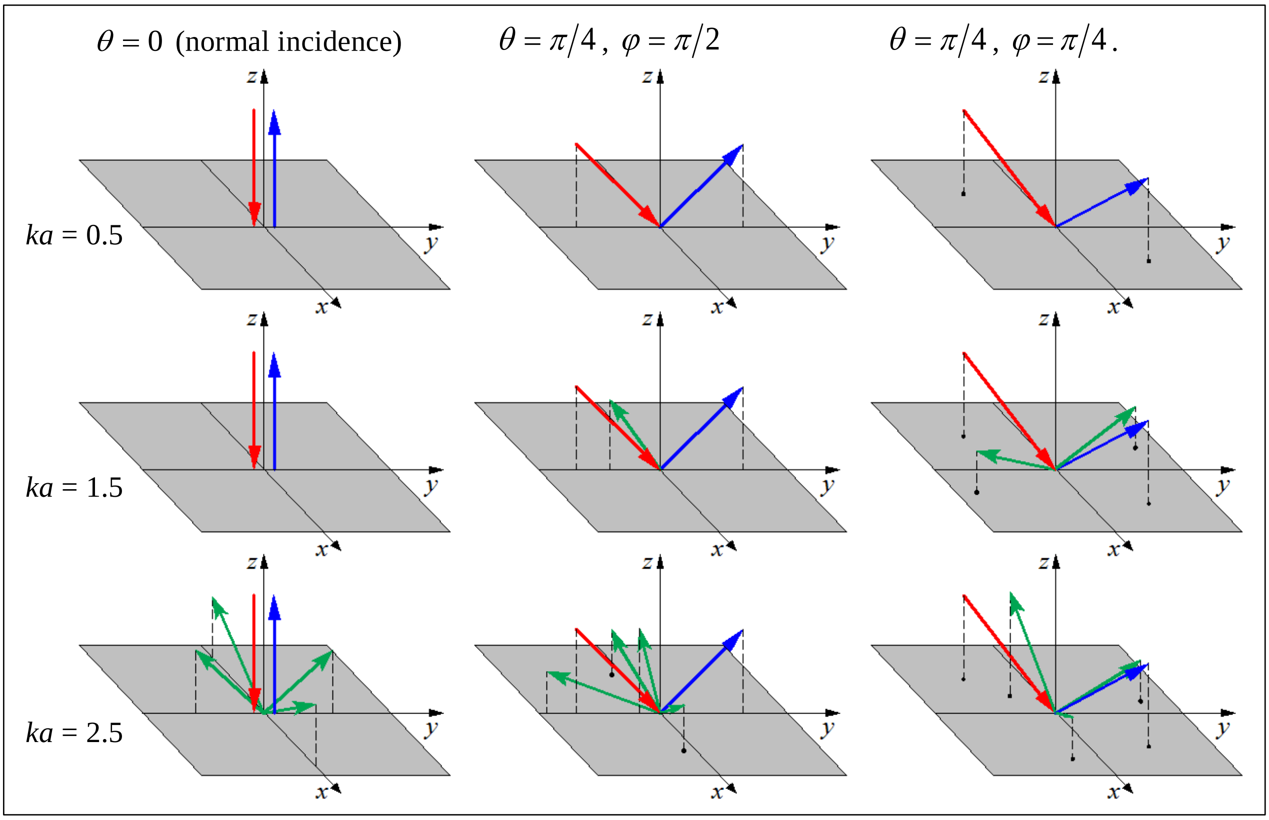

Figure 3. The primary and secondary plane waves: the incident waves (red arrows), the reflected waves and the radiated waves (blue lines), Bragg waves (green lines).

4. Active control strategies

Practical design of the active resonator is based on a loudspeaker (Fig. 1). In order to change acoustic properties of a surface the loudspeakers can be mounted onto it as shown in Fig.4. The regular array of circular pistons is a source of the secondary sound field. If a piston size is much smaller a wavelength and the piston is characterized by the impedance Z_a then the results obtained in the previous paragraph can be applied. The secondary sound field is given by equations (9) and (10). Adjusting the impedance Z_a proposes to control the reflected wave of the primary field. We will analyze three strategies of active control: absorption, specular and diffuse reflection of the incident wave.

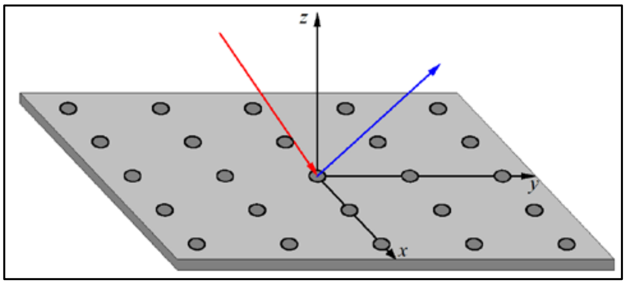

Figure 4. The active resonators on the impedance surface.

4.1. Absorbing array

To obtain the sound field of an array of active resonators placed on the impedance surface we have to pass to the limit $h \to 0$ in (9) and (10). The secondary field is given by

$$P_{II} = -\frac{\omega\rho}{2a^2(Z_a + Z_r)}\big(1 + V(k_z)\big)\sum_{n,m} P_{n,m}\,, \qquad (11)$$

$$P_{n,m} = \frac{1}{k_z^{nm}}\big(1 + V(k_z^{nm})\big)e^{ik_x^n x + ik_y^n y + ik_z^{nm} z}\,. \qquad (12)$$

We see that the wave $P_{0,0}$ coincides with the reflected wave (7). To cancel the reflected wave the following condition has to be satisfied

$$V(k_z) - \frac{\omega\rho}{2a^2(Z_a + Z_r)}\frac{\big(1 + V(k_z)\big)^2}{k_z} = 0\,. \qquad (13)$$

From (13) the impedance of the active resonators providing complete absorption of the incident wave can be found

$$\tilde{Z}_a = \frac{\omega\rho}{2a^2 V(k_z)}\frac{\big(1 + V(k_z)\big)^2}{k_z} - Z_r\,. \qquad (14)$$

The impedance $\tilde{Z}_a$ may be called “optimal” because the active resonators with this impedance give the solution of the active sound field control.

Here we are interested in absorption of the incident wave.

From (14) we can see that the optimal impedance depends on the projection $k_z = k\cos\theta$ of the wave vector. It means that the active resonator array can be adjusted to complete absorption of a wave incident at any fixed angle $\tilde{\theta}$. The waves incident at other angles are not completely absorbed, but the angle $\tilde{\theta}$ can be changed by adjustment of the impedance $\tilde{Z}_a$.

The optimal impedance $\tilde{Z}_a$ depends on the surface impedance Z as well. Obviously application of the active resonators for sound absorption is useful when the surface has low absorbing properties. It takes place if, for example, $\mathrm{Re}\,Z = 0$ or $Z \to \infty$. Last case is typical for hard surfaces and should be considered in details. If $Z >> \rho c$ we can assume that $V(k_z) \approx 1$. The optimal impedance of the active resonators mounted on the hard surface can be found from (21)

$$\tilde{Z}_a = \frac{2\rho c}{a^2 \cos\tilde{\theta}} - Z_r(\tilde{\theta}) . \qquad (15)$$

We see from (18) that in addition to the plane wave $P_{0,0}$ cancelling the reflected wave other waves $P_{n,m}$ can be radiated by the active resonators array. Their directions do not coincide with the primary wave and they cannot be used for active control. Moreover they reduce efficiency of active absorption because they reradiate the sound energy of the incident wave in other directions. Such parasitic radiation does not take place if the spatial period a of the array does not exceed half of the sound wavelength λ, i.e. $a < \lambda/2$. Under this condition sound absorption by the active resonators is the most effective. If $\lambda/2 \le a < \lambda$, radiation of the waves with nonzero values (n,m) depends on the angles of incidence θ and φ. The best case is normal incidence of the primary plane wave. For $\theta = 0$ the active resonator array radiated only the wave $P_{0,0}$ the period a does not exceed the sound wavelength λ, i.e. $a < \lambda$. If $a \ge \lambda$, parasitic radiation takes place in any case.

So the condition $a < \lambda/2$ can be considered as general limitation for application of the active resonators array for active sound control. Sound waves may be absorbed at low frequencies $\omega < \pi c/a$. If the incident angles of the primary wave are limited, for example, if the primary field is anisotropic, it is possible to increase the frequency range up to $\omega < 2\pi c/a$.

If the active resonator array is adjusted to complete absorption of the wave incident at the angle $\tilde{\theta}$, the absorption coefficient of the wave incident at the angle θ can be found from (1), (2), (18) and (22) for $a < \lambda/2$

$$\alpha = 1 - \left(\frac{\cos\theta - \cos\tilde{\theta}}{\cos\theta + \cos\tilde{\theta}} \right)^2 . \qquad (16)$$

Note that the absorption efficiency characterized by α does not depend on the surface impedance Z. Fig. 5a demonstrates the angular dependence of the absorption coefficient of the surface with the impedance $Z/\rho c = 10$ (green line). If the active resonators are adjusted to maximal absorption of normal incidence, i.e. $\tilde{\theta} = 0$, the absorption coefficient of the surface with active resonators is shown by red line. So application of the active resonator array proposed to improve significantly absorbing properties of the surface.

4.2. Reflecting array

Let us consider the surface with high absorbing

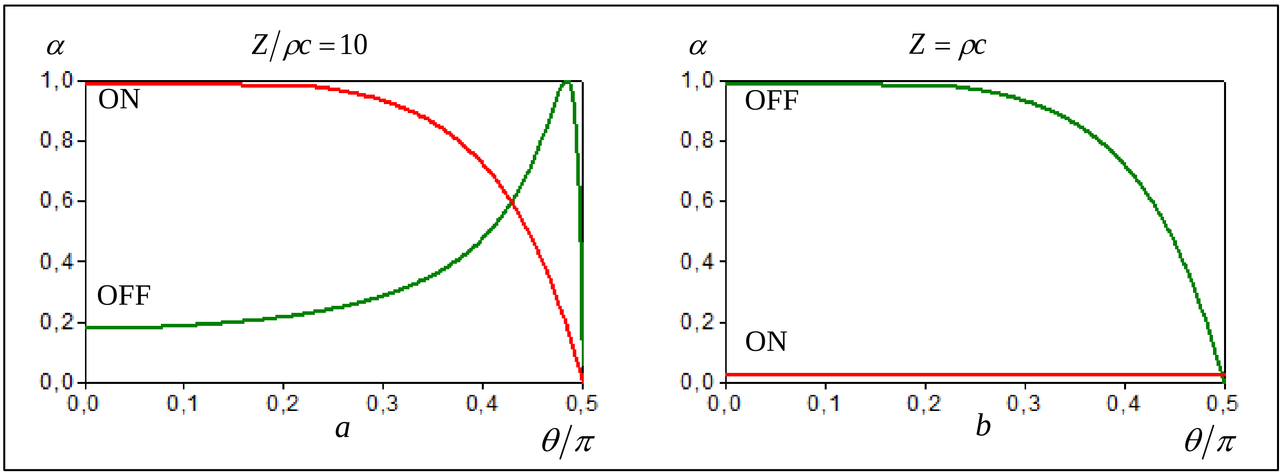

Figure 5. Dependence of the absorption coefficent on the incident angle for maximal absorption (*a*) and maximal reflection (*b*). Absorption coefficient of the surface is marked “OFF”. The surface with the ajusted active reosnators has absorption coefficient marked “ON”.

properties. Usually its impedance should be $|Z| \approx \rho c$. In order to change acoustic properties of the surface the active resonators have to reflect the incident wave. The optimal impedance of the active resonators can be found from previous results.

To simplify the calculation we assume that the surface impedance is real and equal to ρc. Then the reflection coefficient (8) is $V(k_z) = (k_z/k - 1)/(k_z/k + 1)$. The secondary field is given by the equations (11) and (12). The reflection coefficient from the surface with the active resonators is equal to 1 if the amplitude of the superposition of the wave $P_{0,0}$ and the reflected wave (7) is equal to 1. In right part of (13) we have to substitute 1 instead of 0

$$V(k_z) - \frac{\omega\rho}{2a^2(Z_a + Z_r)} \frac{(1 + V(k_z))^2}{k_z} = 1. \tag{17}$$

From (17) the impedance of the active resonators providing complete absorption of the incident wave can be found

$$\tilde{Z}_a = \frac{\omega\rho}{2a^2 k_z} \frac{(1 + V(k_z))^2}{V(k_z) - 1} - Z_r. \tag{18}$$

If the active resonator array has to reflect the wave incident at the angle $\tilde{\theta}$, the optimal impedance (18) can be given by

$$\tilde{Z}_a = -\frac{\rho c}{a^2} \frac{\cos\tilde{\theta}}{1 + \cos\tilde{\theta}} - Z_r(\tilde{\theta}). \tag{19}$$

Note that the real part of the optimal impedance is negative because the active resonators have to compensate the real part of the surface impedance.

Limitation of the frequency range is the same as for the absorbing array. The sound waves may be reflected without parasitic reradiation at frequency range $\omega < \pi c/a$.

Dependence of the absorption coefficient of the surface with impedance $Z = \rho c$ on the incidence angle θ is shown in Fig. 5b. The active resonator array provides total reflection of the incident plane wave for all angles θ. Absorbing properties of the surface are changed to reflective ones due to the active resonators.

4.3. Scattering array

The active resonators applied for absorption or reflection of incident plane waves radiate coherent spherical waves. The spherical waves interfere and generate the plane waves, one of them coincides with the reflected wave and used for active control. Other plane waves are Bragg reflections, they reradiate the incident wave energy and reduce efficiency of active sound field control.

Scattering properties of surfaces are usually provided by means of irregularities. Plane surfaces or curved surfaces with a big radius of curvature do not scatter sound waves.

Application of metastractures proposes to scatter sound waves by means of plane surfaces with small openings [14]. The same effect may be achieved by active resonators. We demonstrate this opportunity by the example of diffuse reflection produced by the plane surfaces with the active resonators.

The active resonators in the regular array as shown in Fig. 4 radiate sound waves coherently because they have the similar impedance. If their impedances are different, sound radiation is not coherent and the secondary field should be isotropic. It means that the field reflected by the surface with the active resonator may be diffuse. So the array of the active resonators with different adjustment may be used for sound scattering.

First of all we consider the simplest case of the absorbing surfaces. It is impedance $|Z| \approx 1$ and reflection coefficient is close to zero for non-glazing incidence. To provide diffuse reflection the sound energy radiated by the active resonators has to be equal to the incident sound energy.

The wave energy density of the incident plane wave (7) is $W_i = 1/2\rho c$. The sound power radiated by the monopole with the volume velocity Q is $w_m = \rho c k^2 Q^2 / 8\pi$. The number of the active resonator per a square unit is $N = a^{-2}$, so the sound power radiated by the square unit of the surface is $w_m N$. The energy reflection coefficient is equal to 1 if $W_i = w_m N$. From this relation we can find the required volume velocity

$$|Q| = \frac{2\sqrt{\pi}a}{\rho\omega}. \tag{20}$$

The volume velocity amplitudes of all active resonators have to be equal for uniform radiation, whereas their phases should be different for incoherent radiation. To satisfy this contortion the impedance of the active resonator placed at the point with coordinates $(x_s y_q, 0)$ has to be equal

$$Z_{s,q} = Z_a e^{if(s,q)}, \tag{21}$$

where Z_a is impedance providing the required volume velocity (20), $f(s,q)$ is a function describing the detuning of the active resonators. In other words all active resonators have the similar impedance $|Z_{s,q}|$, but their phases are different.

Note that the problem of choosing the function $f(s,q)$ has to be considered separately. If the surface is hard, it reflects the sound wave secularly. To provide diffuse reflections the active resonator array has to cancel the reflected wave and to radiate the diffuse field of the same intensity. The optimal impedance for both problems are found and given by equation (14) and (21). These conditions should be combined and, as a result of it, the secondary filed consists of the plane wave canceling the reflected wave of the primary field and the diffuse field.

5. Conclusions

The application of a regular system of active resonators to control the sound reflection properties of surfaces is considered. Each active resonator is characterized by the impedance which can be varied by means of active control. An array of the active resonators mounted onto the surface can absorb or reflect the incident sound waves regardless of the surface impedance. It proposes to vary acoustic properties of the surface.

The most important and perspective active control strategies are presented in Fig. 6. A rigid surface reflects the incidence wave specularly. If the active resonators mounted onto the surface are adjusted to maximal absorption, the incident wave is absorbed by the surface. So active control turns the reflective surface into absorbing one. An alternative strategy is to change the absorbing surface into the reflective one. Also it is possible to provide diffuse reflections due to incoherent radiation. The active resonators have to radiate the sound power equal to the power of the incident wave. If the surface is reflective, the active resonators have to absorb the incident wave as well in order to exclude the specular reflection.

There is a fundamental limitation of the considered method. The incident wave can be effectively controlled if the distance between the active resonators does not exceed a half wavelength. It means that this approach can be applied only for low frequencies. However the frequency band can be varied by selection of the space period of the array.

So use of the active resonators makes it possible to adjust the acoustic properties of the surface. It may be useful for many applications especially for architectural acoustics. But investigation of stability and robustness of the proposed active system is necessarily required.

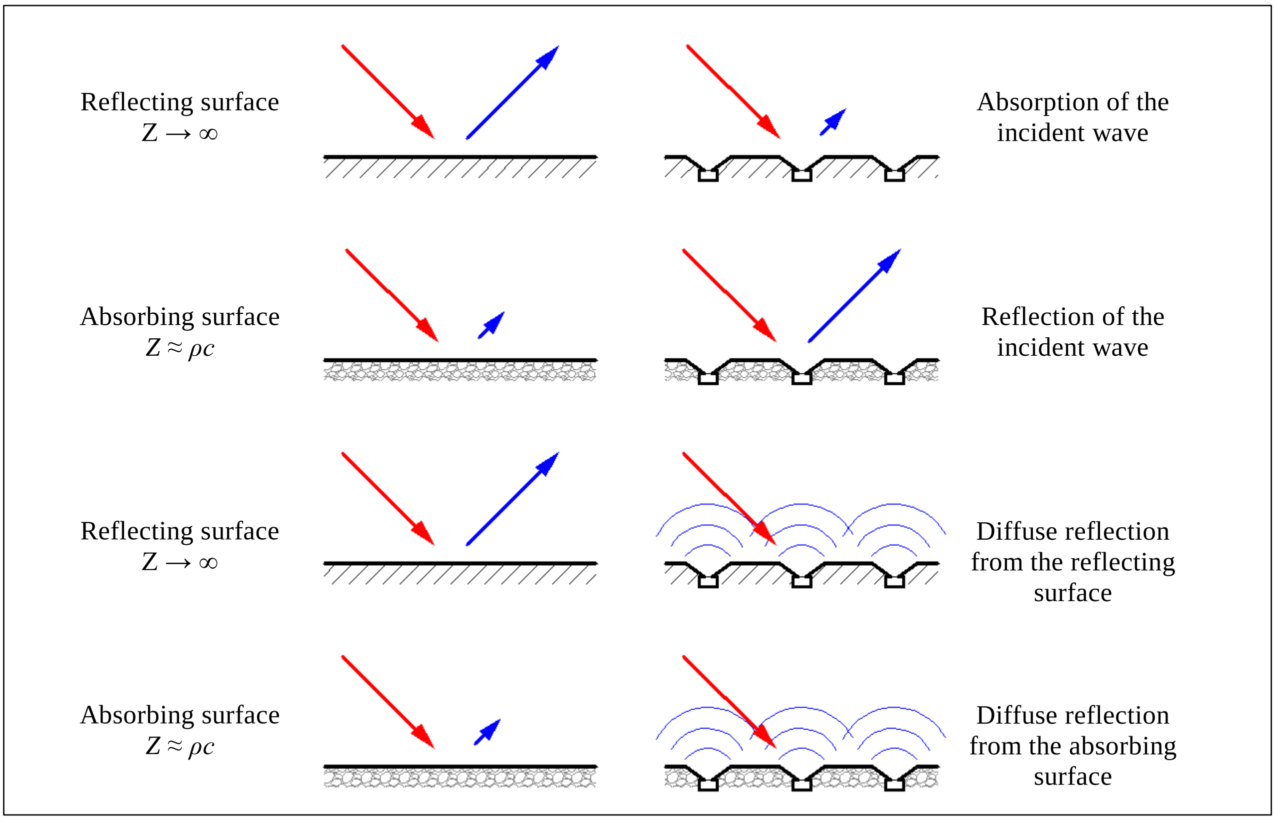

Figure 6. Possible strategies of reflected field control by the active resonators for reflecting and absorbing surfaces.

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