

The use of the bootstrap method for determining sound power level

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Summary

The possibility of using the bootstrap method to determining sound power level for precision, engineering and survey methods was presented in this paper. Minimum values of the bootstrap algorithm input parameters have been determined for estimation of sound power level for each methods. Two independent simulation experiments have been performed for that purpose. The first experiment served to determine the impact of original random sample size, and the second to determine the impact of number of the bootstrap replications on the accuracy of estimation of sound power level. The inference has been carried out based on results of non-parametric statistical tests at significance level $\alpha = 0.05$. The statistical analysis has shown that the minimum size of original random sample n used to estimate the values of sound power level should be 4 elements for precision and survey methods, and 6 elements for engineering method. The minimum number of bootstrap replications necessary for estimation of sound power level should be $B = 5500$. The study on usefulness and effectiveness of the bootstrap method to determination of sound power level in real-life situation was carried out with the use of data representing actual results. The data used to illustrate the proposed solutions and carry out the analysis were results of sound power levels of reference sound power source B&K 4205 were used.

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1. Introduction

Sound power level is one of the main parameters that describe noise source. This parameter is commonly used in acoustics, among other things, to model distribution of equivalent A-weighted sound pressure level in the environment [1, 2] and to noise hazard assessment in the working environment as well as for comparison itself between machines and devices of a certain type [3, 4]. Therefore, the exact value of sound power level is very important. The exact value of this parameter is determined based on precision method for anechoic or hemi-anechoic rooms according to ISO 3745:2012 [5]. In the in-situ conditions use of the precision method to determination of sound power level it is not possible. Therefore in the industrial conditions two methods are used to determine this parameter, i.e. engineering and survey methods according to ISO 3744:2010 [6] and ISO 3746:2010 [7], respectively.

For the reasons mentioned above, it seems to be necessary to implement solutions of non-classical statistic to increase the accuracy of determining sound

power level of industrial devices in the in-situ conditions. These techniques are based on non-parametric statistical methods, allowing to determine the distribution of a random variable without any information on belonging or not to any specific class of distributions and with a limited sample size.

The analysis of papers published in recent years indicates a growing recognition among the researchers for the bootstrap resampling method. It is used with success in point [8, 9] and interval estimation [10, 11] of the noise indicators expected value and uncertainty [12, 13], as well as in planning the measurement strategies [14, 15]. It is often used in statistical analysis of sound measurement results [16, 17].

For these reasons mentioned above, particular attention was paid to the possibility of using the bootstrap resampling method to determining sound power level of noise sources. Discussion of the algorithm, together with an example illustrating its functioning, will be presented further in this paper. The study on usefulness and effectiveness of the bootstrap method to determination of sound power level in real-life situation was carried out with the use of data representing actual results.

2. Assumptions and ideas of the bootstrap method

Consider an observed random sample $\mathbf{x} = (x_1, x_2, \dots, x_n)$ from an unknown probability distribution F with an intent to estimate a parameter of interest $\theta = t(F)$ on the basis of \mathbf{x} . For this purpose, let an estimate $\hat{\theta} = s(\mathbf{x})$ from \mathbf{x} be calculated.

The bootstrap method was introduced in 1979 by B. Efron [18] as a computer-based method for estimating the standard error of $\hat{\theta}$. The bootstrap estimate of standard error requires no theoretical calculations and is available no matter how mathematically complicated the estimator $\hat{\theta} = s(\mathbf{x})$ may be.

Bootstrap methods depend on the concept of a bootstrap sample. Let \hat{F} be the empirical distribution, assigning probability $1/n$ to each of the observed values $x_i, i = 1, 2, \dots, n$. A bootstrap sample is defined as a random sample of size n drawn from \hat{F} , say $\mathbf{x}^b = (x_1^b, x_2^b, \dots, x_n^b)$ [19],

$$\hat{F} \rightarrow (x_1^b, x_2^b, \dots, x_n^b). \quad (1)$$

The symbol “**b**” indicates that \mathbf{x}^b is not the actual data set \mathbf{x} , but rather a resampled version of \mathbf{x} .

Symbolic expression (1) can be also verbalised as follows: the bootstrap data points $x_1^b, x_2^b, \dots, x_n^b$ are a random sample of size n drawn with replacement from the population of n objects (x_1, x_2, \dots, x_n) . The bootstrap data set $(x_1^b, x_2^b, \dots, x_n^b)$ consists of elements of the original data set (x_1, x_2, \dots, x_n) .

Corresponding to a bootstrap data set \mathbf{x}^b is a bootstrap replication of $\hat{\theta}$

$$\hat{\theta}_b = s(\mathbf{x}^b). \quad (2)$$

The quantity $s(\mathbf{x}^b)$ is the result of applying to \mathbf{x}^b the same function $s(\bullet)$ as this applied to \mathbf{x} .

2.1. Point estimation of distribution parameters by bootstrap method

Point estimation of an unknown distribution parameter θ of the examined variable is based on assuming that the estimator value of this parameter at the given sample is its estimation. By applying the Monte Carlo method to the bootstrap, a bootstrap sample B is generated. The bootstrap samples are generated from the original data set (analysed sample). Each bootstrap sample has n elements generated by sampling with replacement n times from the analysed sample. Bootstrap replications $\hat{\theta}_1, \dots, \hat{\theta}_b, \dots, \hat{\theta}_B$ are obtained by calculating the value of the statistics $s(\mathbf{x})$ on each bootstrap sample. The mean of these values can be assumed to be an assessment of parameter θ . Thus, the assessment of parameter θ can be expressed as [19]

$$\bar{\theta}_B = \frac{1}{B} \sum_{i=1}^B \hat{\theta}_b. \quad (3)$$

The bootstrap estimate of the standard error is the standard deviation of the bootstrap replications [19]:

$$\hat{s}_B = \sqrt{\frac{\sum_{b=1}^B (\bar{\theta}_B - \hat{\theta}_b)^2}{B - 1}}. \quad (4)$$

Further, the bootstrap estimate of bias \hat{b}_B based on the B replications is defined by

$$\hat{b}_B = \bar{\theta}_B - \hat{\theta}, \quad (5)$$

where $\bar{\theta}_B$ is bootstrap estimate of parameter θ , and $\hat{\theta}$ is estimate of parameter θ . The value of $\hat{\theta}$ may be calculated from the original sample \mathbf{x} or may differ from $\hat{\theta} = s(\mathbf{x})$, e.g. it determined from the population [19]. Note that both \hat{s}_B and \hat{b}_B can be calculated from the same set of bootstrap replications.

3. Research material

The study on usefulness and effectiveness of the bootstrap method to determination of sound power level in real-life situation was carried out with the use of data representing actual results. The data used to illustrate the proposed solutions and carry out the analysis were results of sound power levels of reference sound power source B&K 4205 were used. The sound power level of this source has been determined using the precision, engineering and survey methods based on measurements of A-weighted sound levels (L_{Aeq}). Measurements of L_{Aeq} were made with a SVAN 959 (SVANTEK, Poland) equipped with SV type preamps and $\frac{1}{2}$ inch free-field 40AN microphone from G.R.A.S. The results of the background noise corrected A-weighted sound levels recorded at each measurement point which has been used to determine the sound power level of source using each method are presented in Figure 1. These data constituted the examined populations with sizes $K = 20$ for precision and engineering methods, and $K = 8$ for survey method.

3.1. Precision method

The precision method for determining the sound power level was based on ISO 3745:2012 [5]. The measurements were carried out in the anechoic room located in the AGH University of Science and Technology, Krakow, Poland. The $K = 20$ measurement points were located on a hemispherical measurement surface with a radius of $r = 2$ m in a hemi-free field according to Table E.1 in Annex E of ISO 3745:2012. The measurement results recorded on February 14th, 2018 in the following meteorological conditions:

- relative humidity: $RH = 24\%$,
- temperature: $t = 19.7^\circ\text{C} = 292.85\text{ K}$,
- static pressure: $p_s = 101.6\text{ kPa}$.

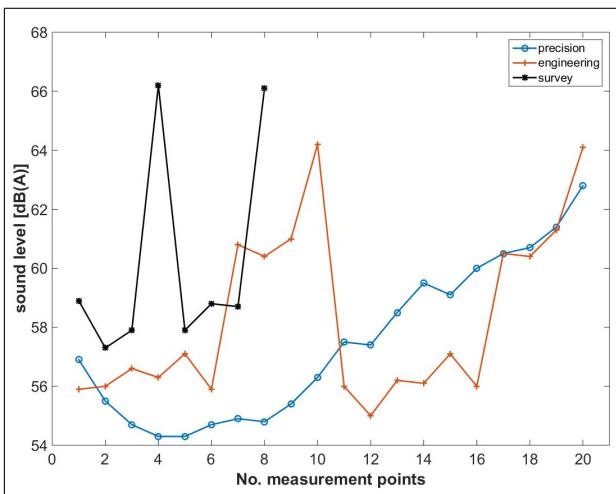


Figure 1. The background noise corrected A-weighted sound levels for each method.

On the grounds of the recorded A-weighted sound levels, value of sound power level for precision method L_{Wpr} is defined by [5] as

$$L_{Wpr} = \overline{L_{pc}} + 10 \log \left(\frac{S}{S_0} \right) + C_1 + C_2 \quad [\text{dB(A)}], \quad (6)$$

where $\overline{L_{pc}}$ is the background noise corrected surface sound pressure level in dB(A), $S = 2\pi r^2$ is the area of the hemispherical measurement surface in m^2 , $S_0 = 1 \text{ m}^2$, C_1 is the reference quantity correction in dB(A), C_2 is the acoustics radiation impedance correction in dB(A). The correction factors C_1 and C_2 are defined in equation (14) of ISO 3745:2012.

The obtained value of sound power level in accordance with (6) is $L_{Wpr} = 72.0 \text{ dB(A)}$.

3.2. Engineering method

The engineering method for determining the sound power level was based on ISO 3744:2010 [7]. The measurements were made on the asphalt playing field of the Plaszowianka Krakow club, Poland. The $K = 20$ measurement points were located on a hemispherical measurement surface with a radius of $r = 2 \text{ m}$ in an essentially free field over a reflecting plane according to Table B.1 in Annex B of ISO 3744:2010. At a distance of 20 m from the measuring surface there were no sound reflecting surfaces. The measurement results recorded on February 17th, 2018 from 11:05 a.m. to 01:15 p.m. in the following meteorological conditions:

- relative humidity: $RH = 75 \%$,
- temperature: $t = 2.8 \text{ }^\circ\text{C} = 275.95 \text{ K}$,
- static pressure: $p_s = 102.4 \text{ kPa}$,
- wind speed: $v = 1 - 2.6 \text{ m/s}$
- wind direction: S – SW.

Based on the measurement results of the A-weighted sound levels, value of sound power

level for engineering method L_{Wen} is defined by [6] as

$$L_{Wen} = \overline{L_p} - K_{1A} - K_{2A} + 10 \log \left(\frac{S}{S_0} \right) + C_1 + C_2 \quad [\text{dB(A)}], \quad (7)$$

where $\overline{L_p}$ is the surface sound pressure level in dB(A), K_{1A} is the background noise correction in dB(A) is defined in equation (16) of ISO 3744:2010, K_{2A} is the environmental correction factor in dB(A) according to Annex A or subsection 4.3.1 of ISO 3744:2010, $S = 2\pi r^2$ is the area of the hemispherical measurement surface in m^2 , $S_0 = 1 \text{ m}^2$, C_1 is the reference quantity correction in dB(A), C_2 is the acoustics radiation impedance correction in dB(A). The correction factors C_1 and C_2 are calculated according to Annex G of ISO 3744:2010.

The obtained value of sound power level in accordance with (7) is $L_{Wen} = 72.6 \text{ dB(A)}$.

3.3. Survey method

The survey method for determining the sound power level was based on ISO 3746:2010 [7]. The measurements were made on the paved parking of the Tauron Arena Krakow, Poland. The $K = 8$ measurement points were located on a hemispherical measurement surface with a radius of $r = 2 \text{ m}$ over a reflecting plane according to Table B.1 in Annex B of ISO 3746:2010. At a distance of 25 m from the measuring surface there were no sound reflecting surfaces. The measurement results recorded on February 17th, 2018 from 08:00 p.m. to 08:40 p.m. in the following meteorological conditions:

- relative humidity: $RH = 81 \%$,
- temperature: $t = 1.0 \text{ }^\circ\text{C} = 274.15 \text{ K}$,
- static pressure: $p_s = 102.2 \text{ kPa}$,
- wind speed: $v = 0.5 - 1 \text{ m/s}$
- wind direction: N.

The value of sound power level for survey method L_{Wsu} was also determined based on equation (7), where $\overline{L_p}$ is the surface sound pressure level in dB(A), K_{1A} is the background noise correction in dB(A) is defined in equation (15) of ISO 3746:2010, K_{2A} is the environmental correction factor in dB(A) according to Annex A or section 4 of ISO 3746:2010, $S = 2\pi r^2$ is the area of the hemispherical measurement surface in m^2 , $S_0 = 1 \text{ m}^2$, C_1 is the reference quantity correction in dB(A), C_2 is the acoustics radiation impedance correction in dB(A). The correction factors C_1 and C_2 are calculated according to Annex G of ISO 3744:2010.

The obtained value of sound power level in accordance with (7) is $L_{Wsu} = 75.0 \text{ dB(A)}$.

4. Simulation experiments, results and discussion

Two simulation experiments have been conducted in order to specify the minimum size of the bootstrap

algorithm input parameters, i.e. original sample size n , and the number of bootstrap replications B in order to determine the expected values of the sound power level with required accuracy.

4.1. Experiment #1

The first experiment served to determine the impact of original random sample size n on the estimation accuracy of sound power level. For that reason, 1000 random samples with sizes $n = 2, 3, \dots, K$ were drawn from the examined population. The original random sample size n simulates the number of measurement points based on which the sound power level is determined. In order to eliminate the impact of the number of bootstrap replications B on the estimation result of the expected value of sound power level, the reconstruction of probability distributions was performed based on the same number of replications B for each sample with size n . The distributions were determined based on $B = 10000$ replications, thus receiving 1000 bootstrap probability distributions with 10000 elements for each original sample size n . Each distribution was used to determine the bootstrap estimate of the expected value of sound power level. The result was 1000-element probability distributions of sound power level which were subjected to further statistical analysis.

First, the Kruskal-Wallis non-parametric test has been performed at the significance level $\alpha = 0.01$ in order to check if there are statistically significant differences in estimated sound power level for various original sample sizes. The test gave the probability values of $p = 3.67 \times 10^{-6}$ and $p = 7.57 \times 10^{-30}$ and $p = 2.46 \times 10^{-13}$ for data from precision, engineering and survey methods, respectively. These values are much less than the assumed level of significance which proves the existence of statistically significant differences in values of estimated parameter. The Tukey-Kramer multiple comparison test at the level of significance $\alpha = 0.05$ was conducted in order to find out between which groups there are differences. The results of the Tukey-Kramer test indicate the original random sample size n based on which the estimated expected values of sound power level are statistically different at the assumed level of significance. The statistical analysis has shown that the minimum size of original sample n used to estimate sound power level should be $n = 4$ for precision and survey methods, while $n = 6$ for engineering method.

The dispersion of obtained results was also analysed by determining the 95% confidence intervals using the percentiles of the bootstrap distribution for each probability distribution obtained using the original random sample of size n . The 95% interval width ($IW_{95\%}$) was defined as

$$IW_{95\%} = |p_{97.5} - p_{2.5}| \quad [\text{dB(A)}], \quad (8)$$

where $p_{2.5}$ and $p_{97.5}$ are the 2.5th and 97.5th empirical percentiles of the bootstrap distribution of sound power level.

Interval widths obtained for precision method fall into ranges from 0.03 dB(A) to 7.12 dB(A) for the original random sample of size $n = 20$ and $n = 2$, respectively. However, for engineering method fall into ranges from 0.03 dB(A) to 6.98 dB(A) for the original random sample of size $n = 20$ and $n = 2$, respectively. On the other hands the $IW_{95\%}$ obtained for survey method fall into ranges from 0.06 dB(A) to 8.51 dB(A) for the original random sample of size $n = 8$ and $n = 2$, respectively. The results clearly show that the dispersion decreases when the size of original random sample increases.

4.2. Experiment #2

This experiment was similar to the first experiment. One thousand original samples each were randomly drawn from the examined populations for each analysed size from 2 to K elements. Then, based on these original samples generated were B bootstrap samples from the interval from 100 to 10000 with an increment of 100. Thus, were obtained 1000-element L_{WA} probability distributions for each analysed number of bootstrap replications in each set which were then further statistically processed.

Similarly, firstly Kruskal-Wallis test was performed for each dataset at the level of significance $\alpha = 0.01$ in order to check if there were statistically significant differences in estimated sound power level depending on the number of bootstrap replications B . The probability values in all analysed sets were higher than the assumed level of significance α : for precision method they were in the 0.07 to 0.99 range, for engineering method in the 0.02 to 0.75 range, and for survey method in the 0.02 to 0.90 range. The results clearly show that in all analysed sets the estimates of expected value of sound power level are not statistically different regardless of the number of bootstrap replications which was used to determine them.

The convergence of the bootstrap algorithm towards the expected value of sound power level has been also analysed in the function of number of bootstrap replications based on the mean value of cumulative sums of bootstrap estimates $M_{CSE,j}$ described by the relationship

$$M_{CSE,j} = \frac{1}{j} \sum_{i=1}^j \bar{\theta}_{B,i} \quad [\text{dB(A)}], \quad (9)$$

for $j = 1, 2, \dots, N$, where N is the sample size ($N = 100$), and $\bar{\theta}_{B,j}$ is a bootstrap estimate of the expected value of sound power level. The number of bootstrap replications at which the algorithm has stabilized, for both expected value and $IW_{95\%}$, is presented in Table I. The analysis of values included in

Table I. Minimum number of bootstrap replications that guarantees the stability of bootstrap algorithm for different sizes n of the original random samples for estimation expected value and $\text{IW}_{95\%}$ of L_{WA} .

method	number of bootstrap replications $B \times 10^3$																		
	original sample size n																		
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
expected value																			
precision	1.4	3.5	4.0	1.0	5.5	0.7	0.5	0.5	1.0	1.1	0.2	3.0	1.0	0.6	1.6	1.0	0.7	2.0	0.7
engineering	3.7	3.0	1.4	3.7	3.6	0.6	1.8	3.5	2.3	3.2	3.1	2.1	2.2	1.1	2.2	0.5	0.4	0.7	0.3
survey	2.8	4.0	5.0	1.7	1.8	2.0	0.6	—	—	—	—	—	—	—	—	—	—	—	—
$\text{IW}_{95\%}$																			
precision	0.9	1.7	0.6	4.7	2.0	0.5	1.4	2.9	2.4	3.3	2.8	1.8	2.5	0.6	3.0	3.4	1.5	2.0	1.7
engineering	1.6	2.8	4.8	5.0	4.6	5.0	3.2	4.8	1.7	1.7	1.3	4.7	1.7	1.4	2.9	3.7	3.0	2.5	2.5
survey	3.9	5.1	3.3	3.8	3.3	3.0	3.0	—	—	—	—	—	—	—	—	—	—	—	—

Table II. Expected value of $\text{IW}_{95\%}$ of L_{WA} of different sizes n of the original random sample.

method	expected value of $\text{IW}_{95\%}$ [dB(A)]																		
	original sample size n																		
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
precision	6.9	6.0	5.1	4.5	3.9	3.5	3.2	2.8	2.6	2.3	2.1	1.9	1.7	1.5	1.3	1.0	0.8	0.6	0.05
engineering	7.3	6.4	5.8	5.4	4.6	4.0	3.8	3.3	3.1	2.7	2.5	2.2	2.0	1.8	1.5	1.2	0.9	0.6	0.06
survey	8.5	6.8	5.8	5.0	4.4	1.7	0.1	—	—	—	—	—	—	—	—	—	—	—	—

Table I indicates that the number of bootstrap replications B at which the algorithm has been considered stable was different depending on the size of the original random sample on which the estimation was based. The results do not show any trend which could indicate any relationship between the required number of bootstrap replications B depending on the size of original random sample in order to stabilize the algorithm. These are random values which depend on the structure of the examined population. The number of bootstrap replications B at which the algorithm was stable is from 200 to 5500 for precision method, and from 300 to 3700 for engineering method, while from 600 to 5000 for survey method.

The next parameter analysed for each dataset was $\text{IW}_{95\%}$ of L_{WA} in the function of the number of bootstrap replications. The $\text{IW}_{95\%}$ was defined identically as in Experiment #1 on the basis of expression (8). The results do not show any trend which could indicate any relationship between $\text{IW}_{95\%}$ of L_{WA} and the number of bootstrap replications. The 95% interval widths oscillate around some set values, that is expected values of $\text{IW}_{95\%}$ of L_{WA} which are included in Table II. The values are from 0.05 dB(A) to 6.9 dB(A) for precision method, and from 0.06 dB(A) to 7.3 dB(A) for engineering method, while from 0.1 dB(A) to 8.5 dB(A) for survey method. The analysis of values in Table II indicates that the $\text{IW}_{95\%}$ is inversely proportional to the size of original random sample based on which the sound power level was estimated.

The convergence of the bootstrap algorithm towards the expected value of 95% interval widths of sound power level has been analysed in the function of

the number of bootstrap replications (Table I) based on the mean value of cumulative sums of 95% interval widths

$$M_{\text{CSIW},j} = \frac{1}{j} \sum_{i=1}^j \text{IW}_{95\%,j} \quad [\text{dB(A)}], \quad (10)$$

for $j = 1, 2, \dots, N$, where N is the sample size ($N = 100$), and $\text{IW}_{95\%,j}$ is an 95% interval width of L_{WA} determined based on the expression (8). Similarly to the algorithm convergence towards the expected value of L_{WA} , there is no trend which could indicate any relationship between the required number of bootstrap replications B depending on the size of original random sample in order to stabilize the algorithm. The number of bootstrap replications at which the algorithm has been stabilized ranges from 500 to 4700 for precision method, and from 1300 to 5000 for engineering method, while from 3000 to 5100 for survey method.

Based on the presented results of algorithm convergence, it was concluded that the minimum number of bootstrap replications for estimation of the expected value and 95% interval widths of sound power level should be $B = 5500$ in order to ensure an adequate algorithm convergence and consequently a satisfactory accuracy of estimated statistics.

5. CONCLUSIONS

The paper determines the minimum size of the bootstrap algorithm parameters (size of the original random sample and the number of bootstrap replications) necessary to estimate the expected value and the 95%

confidence interval with required accuracy for different methods of determining sound power level. To this end, two independent simulation experiments were conducted. Experiment #1 served to determine the size of the original random sample, and Experiment #2 was used to determine impact of the number of bootstrap replications on the estimation accuracy of sound power level and its 95% confidence interval.

The statistical analysis was carried out on the basis of Kruskal-Wallis test. Next, multiple comparison procedures were used for pairwise comparisons between the means using non-parametric Tukey-Kramer test at significance level $\alpha = 0.05$.

The statistical analysis has shown that the minimum size of original random sample n used to estimate the values of sound power level should be 4 elements for precision and survey methods, and 6 elements for engineering method.

The estimates of sound power level do not have a statistically significant difference regardless of the number of bootstrap replications B based on which they were determined.

The minimum number of bootstrap replications necessary to estimate the expected value and 95% confidence interval of sound power level should be $B = 5500$ in order to ensure an adequate algorithm convergence and consequently a satisfactory accuracy of estimated statistics.

The results of both experiments clearly indicate that 95% interval width decreases as the original random sample grows, thus proving a very good stability of the bootstrap algorithm and confirming that this approach can be successfully used to estimate not only sound power level, but also other acoustic parameters.

The numerical experiment results presented in this paper refer only to one sound source. The minimum size of the bootstrap algorithm input parameters can be different for other sound sources. Therefore the minimum size of these parameters used for the determination of sound power level of each other source must be adapted according to the characteristics of this source, because the proposed methodology may be applied to other types of noise sources.

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